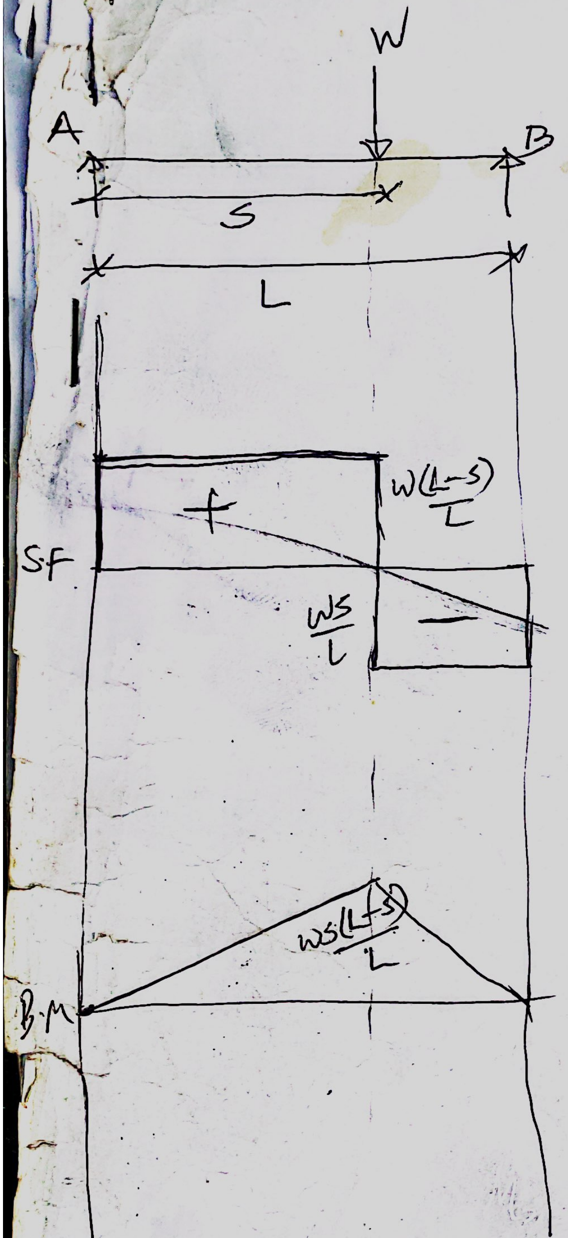
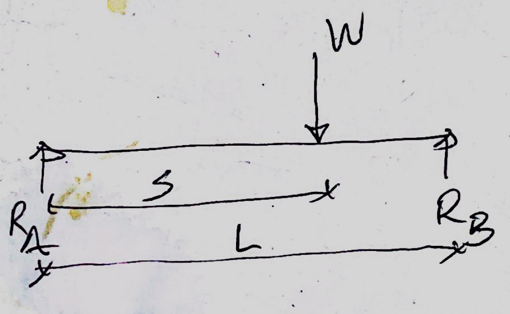
Example 1 Solution





Find the Rxns A & B

$\Sigma F_y = 0$

$R_A + R_B = W$ —— (1)

$\Sigma M_A = 0$

$-R_B L + Ws = 0$ —— (2)

$R_B L = Ws$

$R_B = \frac{Ws}{L}$ —— (2)

from eqn 1

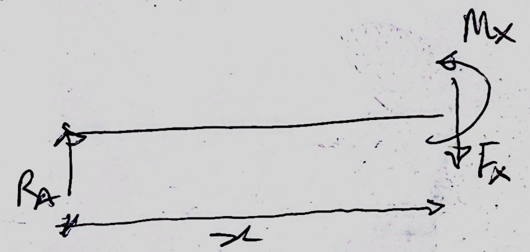
$R_A = W - R_B$

$= W - \frac{Ws}{L}$

$= W\left(1 - \frac{s}{L}\right)$ —— 3a

$= \frac{W(L-S)}{L}$ —— 3b

Section 1 $0 \le x \le s$



SF

$\Sigma F_y = 0$

$R_A - F_x = 0$

$F_x = R_A$

$= \frac{W(L-S)}{L}$

This is the general eqn of S.F. for the section. Constant, not func. of x

at $x = 0$; $F_0 = \frac{W(L-S)}{L}$

at $x = S$ $F_s = \frac{W(L-S)}{L}$

BM

$\Sigma M_s = 0$

$R_A x - M_x = 0$

$M_x = R_A x$

$= \frac{W(L-S)x}{L}$

this is BM eqn. It is a linear expression in x

at $x = 0$

$$M_{x=0} = \cancel{R_A}\,\frac{W(L-S)}{L}(0)$$

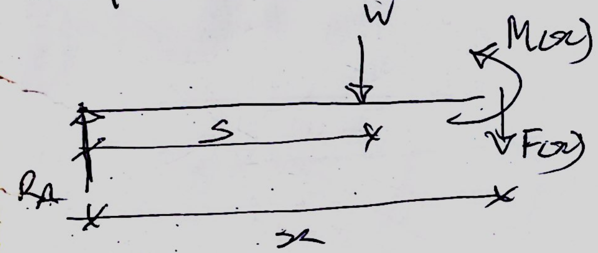
$$= 0$$

at $x = S$

$$M_{x=S} = \frac{W(L-S)S}{L}$$

$$= \frac{WS(L-S)}{L}$$

Section 2 → $S < x \le L$



S.F

$$\Sigma F_y = 0$$

$$R_A - W - F_x = 0$$

$$F_x = \cancel{W - R_A}\; R_A - W$$

$$= \frac{W(L-S)}{L} - W$$

$$=$$

$$F_x = -\frac{WS}{L}$$

this is expression. It is the

at $x = 0$

$$F_x = -\frac{WS}{L}$$

$$F_{x=L} = -\frac{WS}{L}$$

BM

$$R_A x - W(x-S) - M_{(x)} = 0$$

$$M_x = R_A x - W(x-S)$$

$$= \frac{W(L-S)}{L}x - W(x-S)$$

$$= \cancel{Wx} - \frac{WSx}{L} - \cancel{Wx} + WS$$

$$= -\frac{WSx}{L} + WS$$

this is the BM expression

at $x = S$

$$M_{(x=S)} = -\frac{WSS}{L} + WS$$

$$= WS\left(1 - \frac{S}{L}\right)$$

$$= WS\frac{(L-S)}{L}$$

at $x = L$

$$M_{(x=L)} = -\frac{WSL}{L} + WS$$

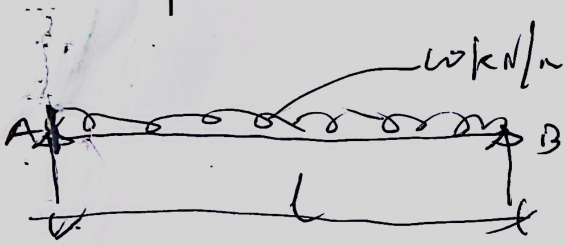
$$= -WS + WS$$

$$= 0 \text{ kNm}$$

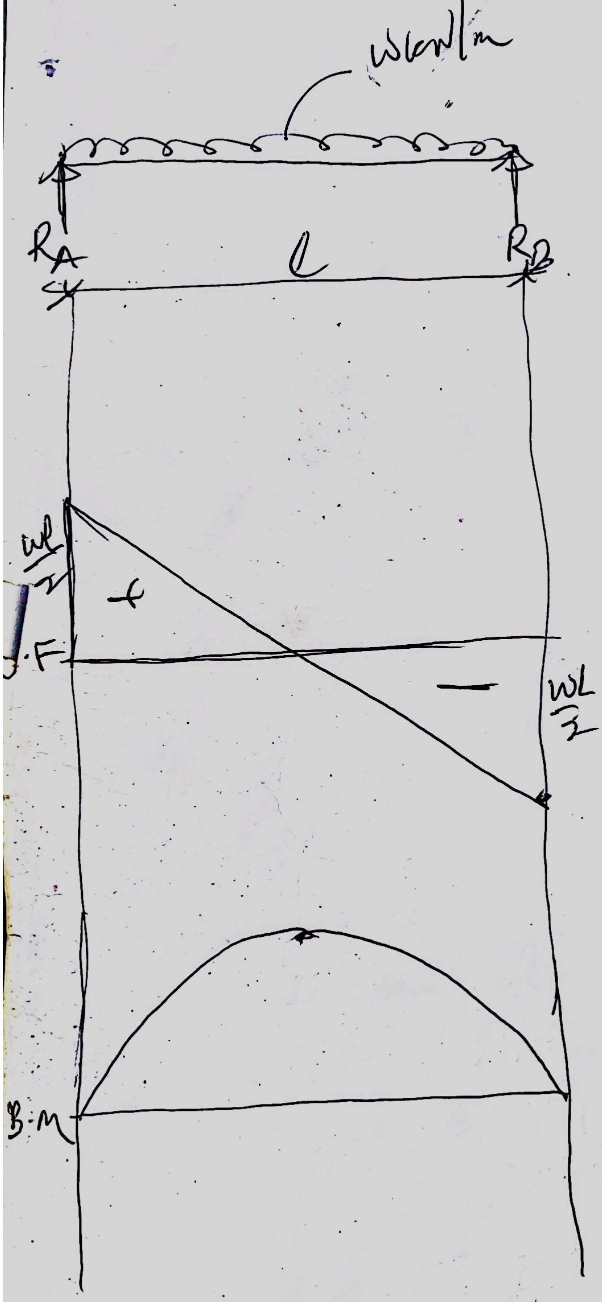
Conclusion

For simply supported beam with point load,

Example 2



Solution



Total load = wL kN

To find R_A & R_B

$\Sigma F_y = 0$

$R_A + R_B = wL$ —— (1)

ΣM_B (or A)

$-R_B L + wL\cdot\frac{L}{2} = 0$

$R_B L = \frac{wL^2}{2}$

$R_B = \frac{wL}{2}$ kN —— (2)

from eqn 1

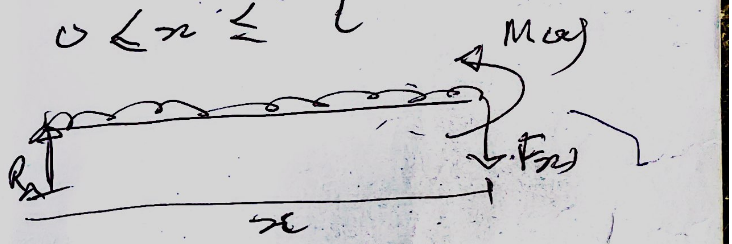
$R_A = wL - R_B$

$= wL - \frac{wL}{2}$

$= \frac{wL}{2}$ —— (3)

Just one section is necessary

$0 \le x \le L$



S.F

$\Sigma F_y = 0$

$R_A - wx - F_x = 0$

$F_x = R_A - wx$

$= \frac{wL}{2} - wx$ —— (4)

this is a general eqn ...

at $x = 0$

$F_{(x=0)} = \frac{wL}{2} - 0$

$= \frac{wL}{2}$ kN

at $x = L$

$F_{(x=L)} = \frac{wL}{2} - wL$

$= -\frac{wL}{2}$ kN

B.M

$\Sigma M_x = 0$

$R_A x - Wx \cdot \frac{x}{2} - M_x = 0$

$M_x = R_A x - \frac{Wx^2}{2}$

$= \frac{WLx}{2} - \frac{Wx^2}{2}$ ——— (2)

This is the B.M. expression & it is quadratic

at $x = 0$

$M_{(x=0)} = \frac{WL(0)}{2} - \frac{W(0)^2}{2}$

$= 0$ kN.m

at $x = L$

$M_{(x=L)} = \frac{WLL}{2} - \frac{WL^2}{2}$

$= 0$ kNm

For a simply supported beam carrying a udl, the relationship between the SF & BM is that at the point of zero shear, the BM is maximum

$F = \frac{dM}{dx} = 0$

$= R_A - Wx$

$= 0$

$x = \frac{R_A}{W} = \frac{\frac{WL}{2}}{W} = \frac{L}{2}$ m

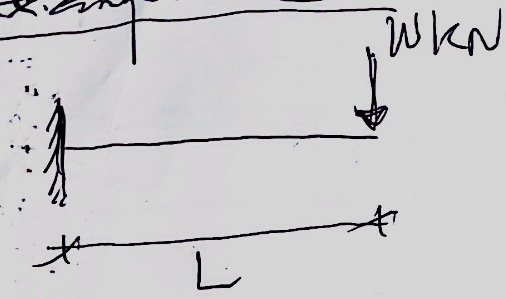
Thus the B.M at $x = \frac{L}{2}$

$$M_{(x=\frac{L}{2})} = \frac{wL}{2}\frac{L}{2} - \frac{w(L/2)^2}{2}$$

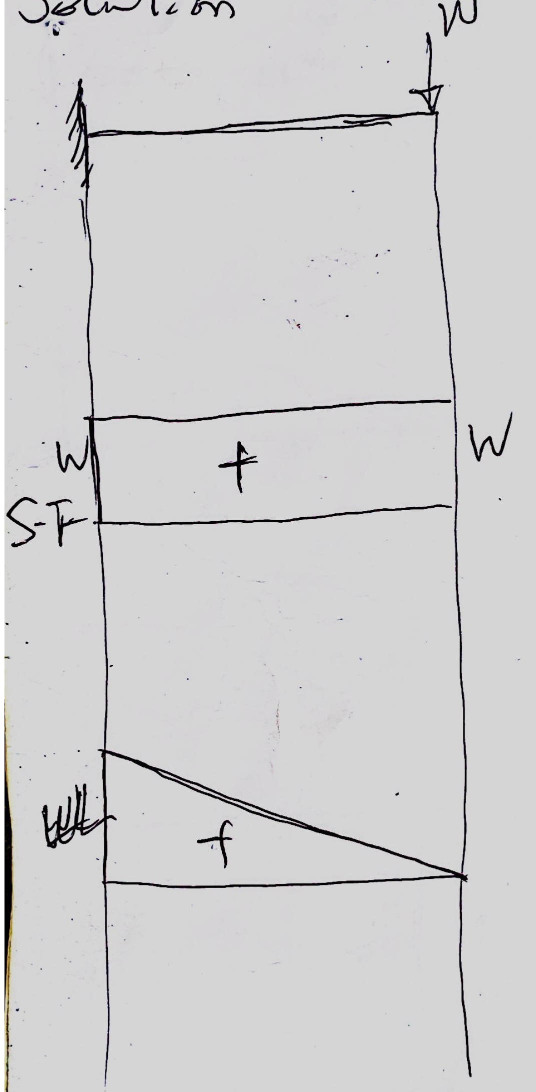
$$= \frac{wl^2}{4} - \frac{wl^2}{8}$$

$$= \frac{wl^2}{8} \text{ kN-m}$$

Example 3

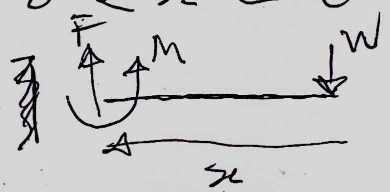


Solution



Only one section will be cut referenced at RHS.

$0 < x < l$



SF

$\sum F_y = 0$

$F - W = 0$

$F_{(x)} = W$ ——— (1)

That is, independent of the beam span. Constant throughout the length

at $x = 0$

$F_{(x=0)} = W$ kN

at $x = l$

$F_{(x=l)} = W$ kN

B.M

$\sum M_x = 0$

$Wx - M_x = 0$

$M_{(x)} = Wx$ ——— (2)

A fcn of x.

at $x = 0$

$M_{(x=0)} = W \times 0$

$= 0$ kN.m

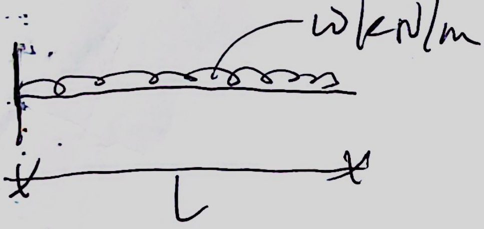
at $x = l$

$M_{(x=l)} = WL$

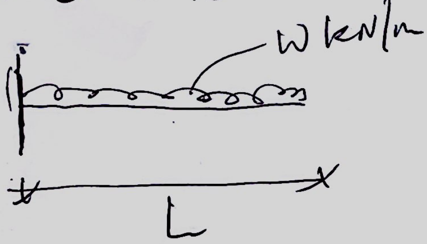
$= Wl$ (kN.m)

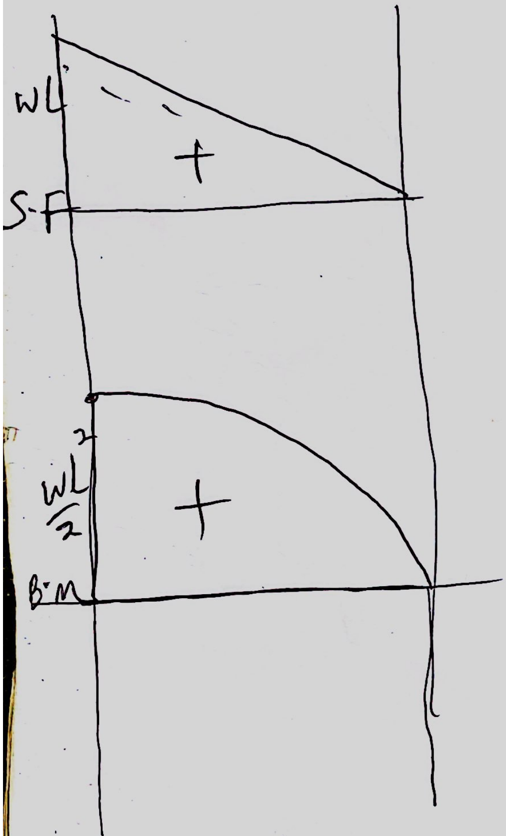
W

Example 4

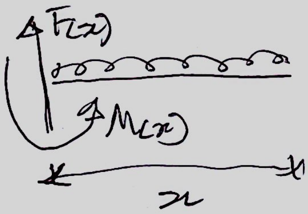


Solution





$0 < x < l$



S.F

$\Sigma F_y = 0 \Rightarrow$

$F(x) - wx = 0$

$F(x) = wx \longrightarrow (1)$

That is a linear function of x

at $x = 0$

$F_{(x=0)} = w \times 0 = 0\,kN$

at $x = L$

$F_{(x=L)} = wL$

$= wL\,kN$

B.M

$\Sigma M_{(x)} = 0$

$-M(x) + wx \cdot \frac{x}{2} \Rightarrow$

$M(x) = \frac{wx^2}{2} \longrightarrow (2)$

A quadratic expression

at $x = 0$

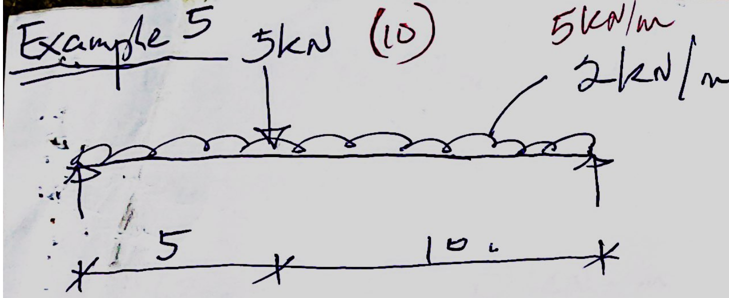
$M_{(x=0)} = 0\,kN.m$

at $x = L$

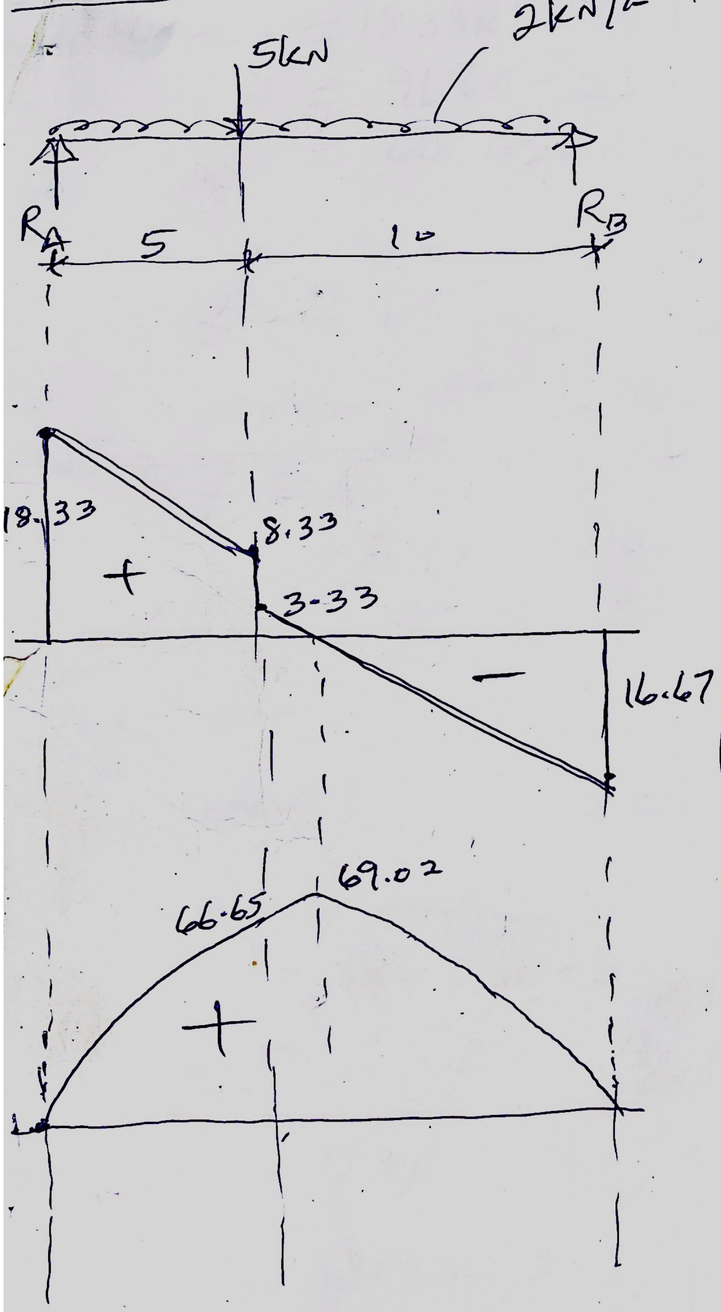
$M_{(x=L)} = \frac{wL^2}{2}$

$= \frac{wL^2}{2}\,kN.m$

Example 5 (10)



Solution



$R_A + R_B = 5 + 2 \times 15$

$= 5 + 30$

$= 35$ ——— (1)

Moment about A

$5 \times 5 + 2 \times 15 \cdot \frac{15}{2} - 15R_B = 0$

$15R_B = 25 + 225 = 250$

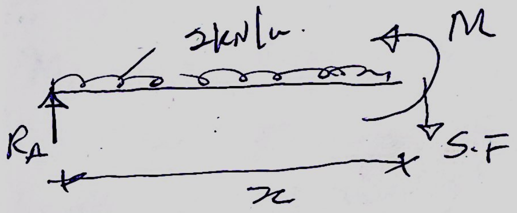
$\underline{R_B = 16.67 \text{ kN}}$

from eqn 1

$R_A = 35 - 16.67$

$\underline{= 18.33 \text{ kN}}$

$0 \leq x \leq 5$



SF

$\Sigma F_y = 0$

$R_A - 2x - F_{(x)} = 0$

$F_{(x)} = R_A - 2x$

$= 18.33 - 2x$ (2)

at $x = 0$

$F_{(x=0)} = 18.33$ kN

at $x = 5$

$F_{(x=5)} = 18.33 - 2 \times 5$

$= 8.33$ kN

BM

$R_A x - 2x \cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - x^2$

Scanned with CamScanner

$\therefore M_{(x)} = 18.33x - x^2$

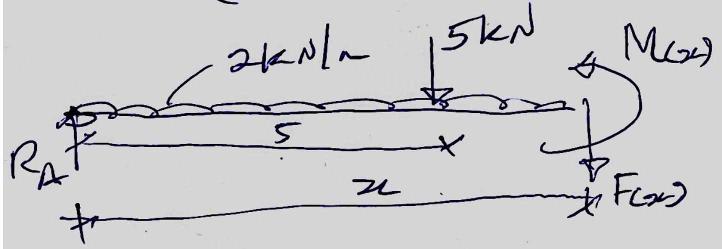
at $x = 0$

$M_{(x=0)} = 0\ kN.m$

at $x = 5$

$$M_{(x=5)} = 18.33\times5 - (5)^2$$
$$= 91.65 - 25$$
$$= 66.65\ kN\text{-}m$$

$5 \leq x \leq 15$



S.F:

$\Sigma F_y = 0$

$$R_A - 2x - 5 - F_{(x)} = 0$$
$$F_{(x)} = R_A - 2x - 5$$
$$= 18.33 - 2x - 5$$
$$= 13.33 - 2x \quad (*)$$

at $x = 5$

$$F_{(x=5)} = 13.33 - 2\times5$$
$$= 13.33 - 10$$
$$= 3.33\ kN$$

at $x = 15$

$$F_{(x=15)} = 13.33 - 2\times15$$
$$= -16.67\ kN$$

At the point where S.F crosses the axis,

$F_{(x)} = 0$

That is

$$0 = 13.33 - 2x$$
$$x = 6.67\ m \text{ from A}$$

BM

$\Sigma M_o = 0$

$$R_A x - 2.x.\frac{x}{2} - 5(x-5) - M_{(x)} = 0$$
$$M_{(x)} = R_A x - x^2 - 5x + 25$$
$$= 18.33x - x^2 - 5x + 25$$
$$= -x^2 + 13.33x + 25$$

at $x = 5$

$$M_{(x=5)} = -(25) + 66.65 + 25$$
$$= 66.65\ kN.m$$

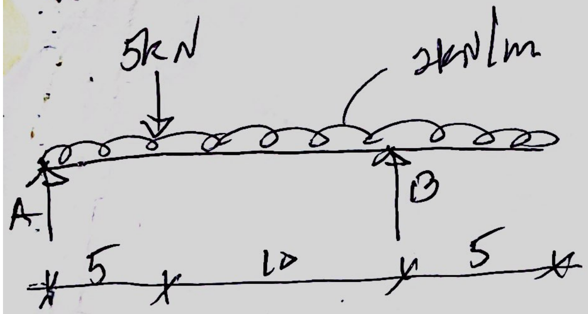
at $x = 15$

$$M_{(x=15)} = -225 + 199.95 + 25$$
$$= 0.05\ kN\text{-}m$$
$$\approx 0\ kN.m$$

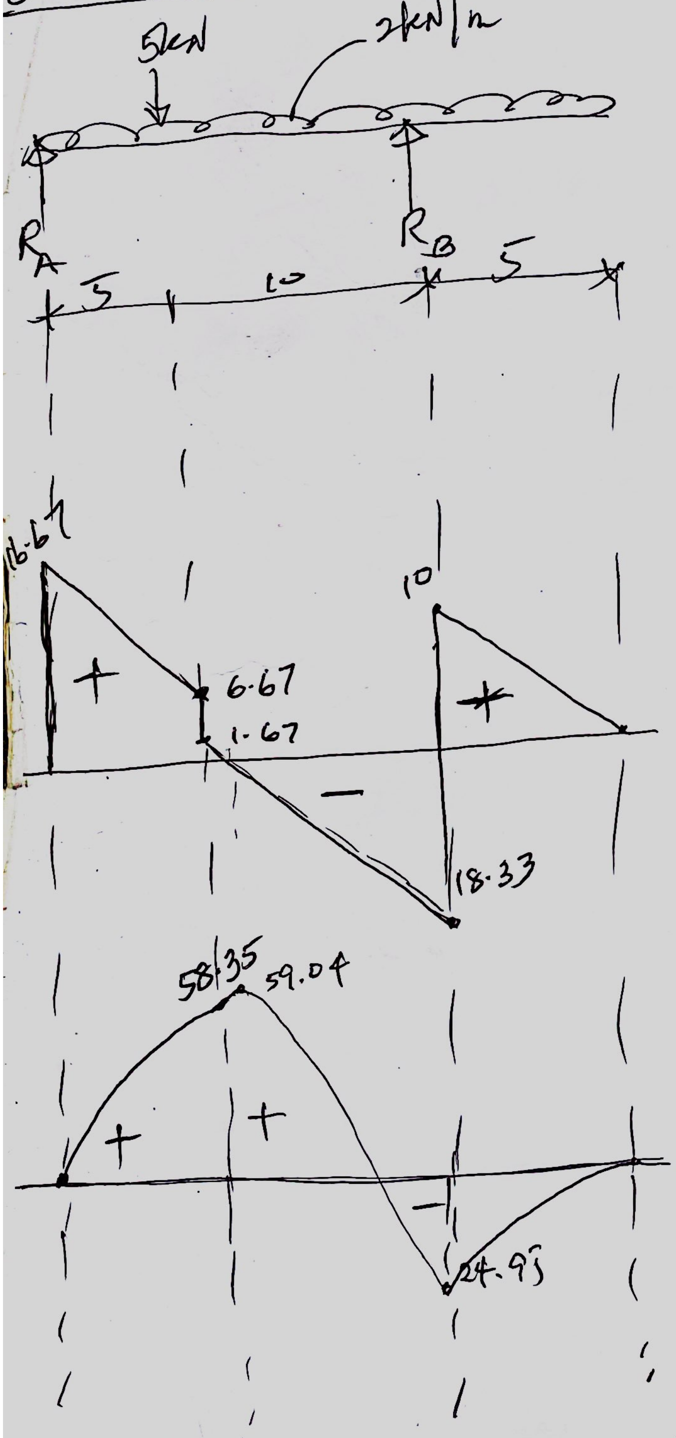
The max Moment at $x = 6.67\ m$

$$M_{max} = -(44.49) + 88.91 + 25$$
$$= 69.42\ kN.m$$

Example 6



Solution



Find the reactions R_A & R_B

$$R_A + R_B - 2\times 20 - 5 = 0$$

$$R_A + R_B = 45 \quad \text{——— (1)}$$

M_A

$$-15R_B + 5\times 5 + 2\times 20 \times \frac{20}{2} = 0$$

$$-15R_B + 25 + 400 = 0$$

$$15R_B = 425$$

$$R_B = 28.33\,KN$$

from eqn 1

$$R_A = 45 - R_B$$

$$= 45 - 28.33$$

$$= 16.67\,KN$$

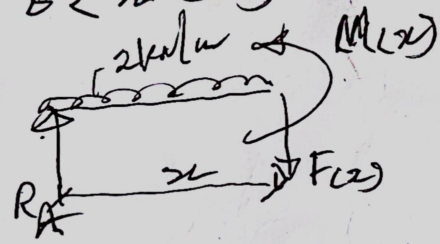
Sections

(1) $0 < x < 5$

(2) $5 < x < 15$

(3) $15 < x < 20$

(1) $0 < x < 5$



S.F.

$$R_A - 2x - F(x) = 0$$

$$F(x) = R_A - 2x$$

$$\boxed{= 16.67 - 2x} \quad (2)$$

at $x = 0\,m$

$$F(x) = \underline{16.67\,KN}$$

$x = 5\,m$

$$F(x) = 16.67 - 2\times 5 = \underline{6.67\,kN}$$

B.M

$R_A x - 2 \cdot x \cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - x^2$

$\boxed{= 16.67x - x^2}$ (3)

at $x = 0$

$M_{(x)} = 0$

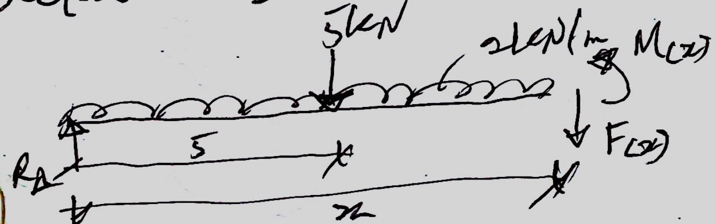
at $x = 5\,m$

$M_{(x)} = 16.67 \times 5 - (5)^2$

$= 83.35 - 25$

$= 58.35\ kN.m$

Section 2 $5 < x < 15$



S.F.

$R_A - 2x - 5 - F_{(x)} = 0$

$F_{(x)} = R_A - 2x - 5$

$= 16.67 - 2x - 5$

$\boxed{= 11.67 - 2x}$ — (4)

at $x = 5\,m$

$F_{(x)} = 11.67 - 10$

$= 1.67\ kN$

$x = 15\,m$

$F_{(x)} = 11.67 - 2 \times 15$

$= 11.67 - 30$

$= -18.33\ kN$

At point where S.F. = 0

eqn 4 = 0

That is

$11.67 - 2x = 0$

$2x = 11.67$

$\boxed{x = 5.84\,m}$ (from the Left)

B.M

$R_A x - 5(x-5) - 2 \cdot x \cdot \frac{x}{2} - M_{(x)} = 0$

$M_{(x)} = R_A x - 5x + 15 - x^2$

$= 16.67x - 5x + 25 - x^2$

$= 11.67x + 25 - x^2$

$\boxed{= -x^2 + 11.67x + 25}$ (5)

at $x = 5\,m$

$-(5)^2$

$M_{(x)} = -25 + 58.35 + 25$

$= 58.35\ kN.m$

at $x = 15\,m$

$M_{(x)} = -(15)^2 + 175.05 + 25$

$= -225 + 175.05 + 25$

$= -24.95\ kN.m$

Max Moment at $x = 5.84$

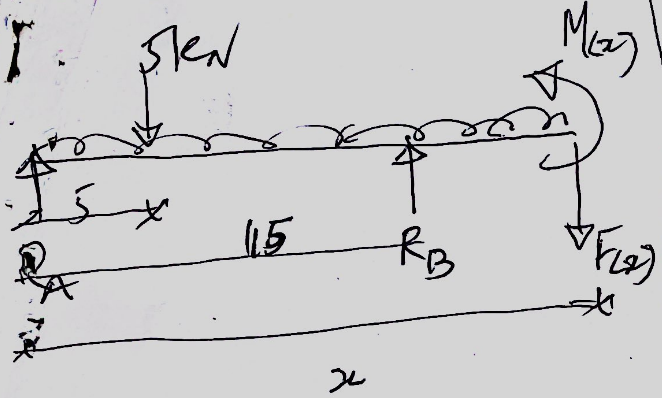
$M_{max} = -(5.84)^2 + 11.67 \times 5.84 + 25$

$= -34.11 + 68.15 + 25$

$\boxed{= 59.04\ kN.m}$

Section 3

$15 < x < 20$



SF..

$R_A - 5 - 2x + R_B - F(x) = 0$

$F(x) = R_A + R_B - 5 - 2x$

$= 16.67 + 28.33 - 5 - 2x$

$= 45 - 5 - 2x$

$\boxed{= 40 - 2x}$

(6)

at $x = 15$ m

$F(x) = 40 - 30$

$= 10$ kN

at $x = 20$ m

$F(x) = 40 - 40$

$= 0$ kN

BM

$R_A x - 5(x-5) - 2 \cdot x \cdot \frac{x}{2} + R_B(x-15) - M(x) = 0$

$R_A x - 5x + 25 - x^2 + R_B x - 15R_B - M(x) = 0$

$x(R_A - 5 + R_B) + 25 - 15R_B - x^2 = M(x)$

$40x + 25 - 15 \times 28.33 - x^2 = M(x)$

12

$40x - 399.95 - x^2 = M(x)$

$M(x) = -x^2 + 40x - 399.95$

at $x = 15$ m

$M(x) = -225 + 600 - 399.95$

$= 24.95$ kN·m

at $x = 20$ m

$M(x) = -(20)^2 + 40 \times 20 - 399.95$

$= -400 + 800 - 399.95$

≈ 0 kN·m