Viscosity

Viscosity may be defined as the property of a fluid which determines its resistance to shearing stresses. It is a measure of the internal fluid friction which causes resistance to flow.

An ideal fluid has no viscosity. However there is no fluid which can be classified as ~~ideal~~ perfectly ideal fluid. However, fluid with very little viscosity are sometimes considered as ideal fluids.

Viscosity of fluids are due to cohesion and interaction between particles. From the figure shown below, when two layers of fluid, at a distance 'dy' apart, moves one over the other at different velocities, say u and u+du, the viscosity together with relative velocity causes a shear stress acting between the fluid layers. The top layer causes a shear stress on the adjacent lower layer while the lower layer causes a shear stress on the adjacent top layer. This shear stress is proportional to the rate of change of velocity with respect to y. It is denoted by τ (tau)

Mathematically $\tau \propto \frac{du}{dy}$

$$\tau = \mu \cdot \frac{du}{dy}$$

where μ = constant of proportionality known as co-efficient of dynamic viscosity or only viscosity.

du/dy = rate of shear stress or rate of shear deformation or velocity gradient

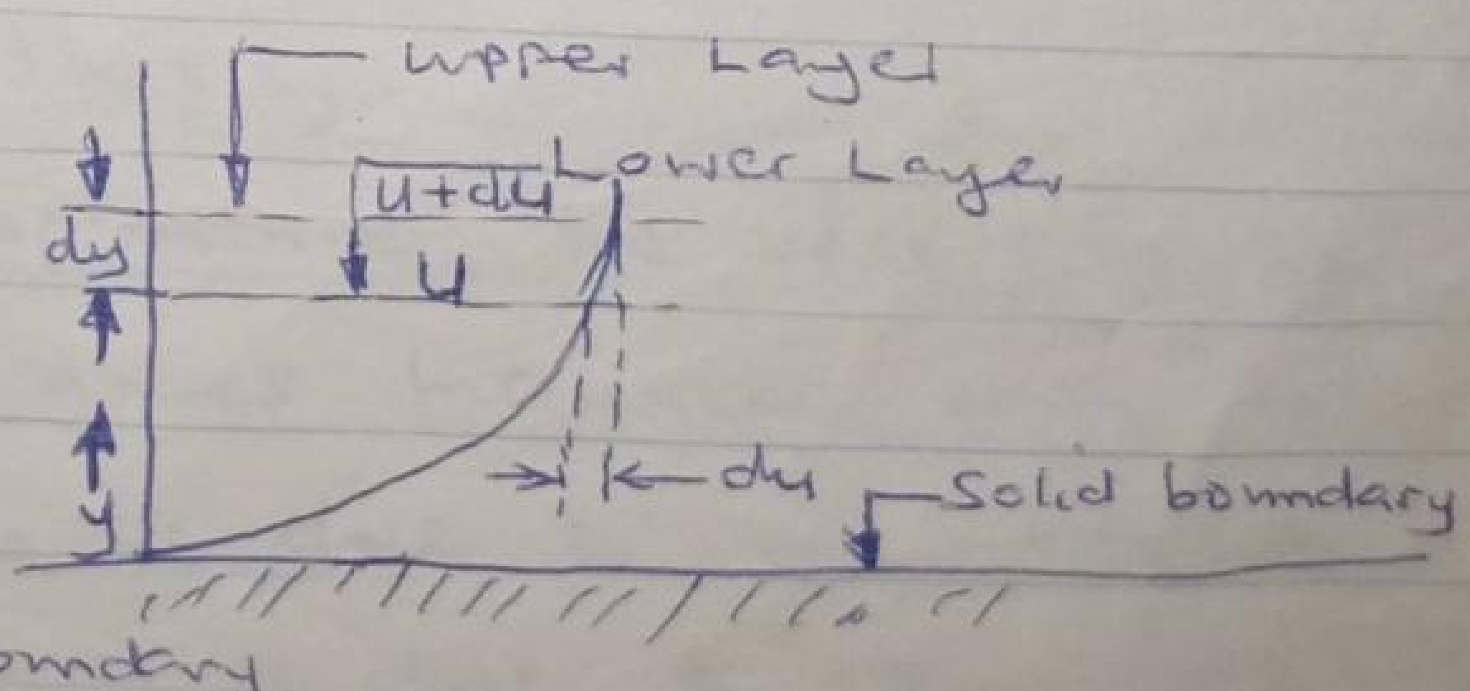


Fig. Velocity variation near a solid boundary

From the figure $\mu = \frac{\tau}{\left[\frac{du}{dy}\right]}$

Hence viscosity can be defined as the shear stress required to produce unit rate of shear strain. Unit of viscosity in S.I unit is ~~Ns~~ $N \cdot s/m^2$ in MKS Units is $Kg_f\,Sec/m^2$ and Poise in CGS.

Kinetic viscosity is defined as the ratio between the dynamic viscosity and density of fluid. It is denoted by ν (called nu)

$$\text{Mathematically } \nu = \frac{\text{viscosity}}{\text{density}} = \mu/\rho$$

S.I unit is m^2/s, MKS m^2/sec and CGS is called stoke (cm^2/sec)

Newton's law viscosity States that the shear stress (τ) on a fluid element layer is directly proportional to the rate of shear strain. The constant of proportionality is called the co-efficient of viscosity.

$$\tau = \mu \, du/dy$$

Fluid that ~~flows~~ follows this law are called Newtonian fluids.

Assignment 1

Draw the ~~graph~~ diagram of Variation of shear stress with gradient of different types of fluid and the relationship between shear stress (τ) and rate of angular deformation for Various types of fluid.

Effect of Temperature on Viscosity.

Viscosity is affected by temperature. The viscosity of liquid decreases but that of gases increases with increasing temperature. This is due to the reason that in liquids the shear stress is due to the

Inter-molecular cohesion which decreases with increase temperature. In gases the inter-molecular cohesion ~~which decreases with increase temperature~~ is negligible and the shear stress is due to exchange of momentum of the molecules, normal to the direction of motion. The molecular activities increases with rise in temperature and so does the viscosity of gas.

Effect of pressure on viscosity

~~The~~ Viscosity under ordinary conditions is not appreciably affected by the changes in pressure. However, viscosity of some oils has been found to increace with increase in pressure.

Surface Tension

Cohesion is the intermolecular attraction between molecules of the same liquid. It enables a liquid to resist small amount of tensile stress. Cohesion is thus the tendency of the liquid to remain as one assemblage of particles

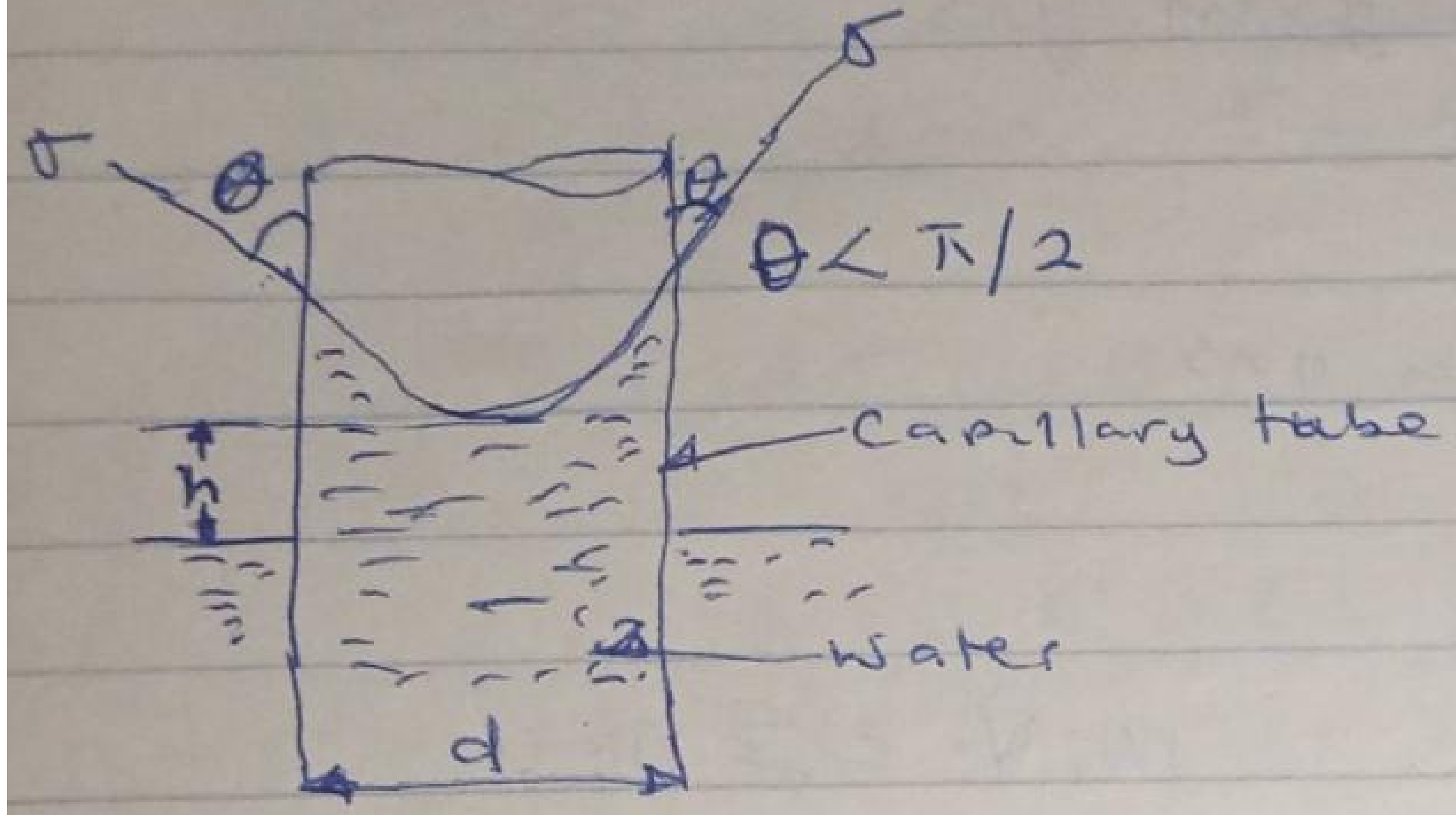
Adhesion is the attraction between the molecules of a liquid and the molecules of a solid boundary surface in contact with the liquid. This property enables a liquid to stick to another body.

Capillary action is due to both cohesion and adhesion

Surface tension

Capillarity

Capillarity is a phenomenon by which a liquid which depends upon its specific gravity rises into a thin glass tube above or below its general level. This phenomenon is due to the combined effect of cohesion and adhesion of liquid particles.



h = Capillary rise

d = diameter of capillary tube

θ = Angle of contact of the water surface

σ = Surface tension force for unit length

W = weight density (ρg)

Upward surface tension force (lifting force) = weight of the water column in the tube (gravity force)

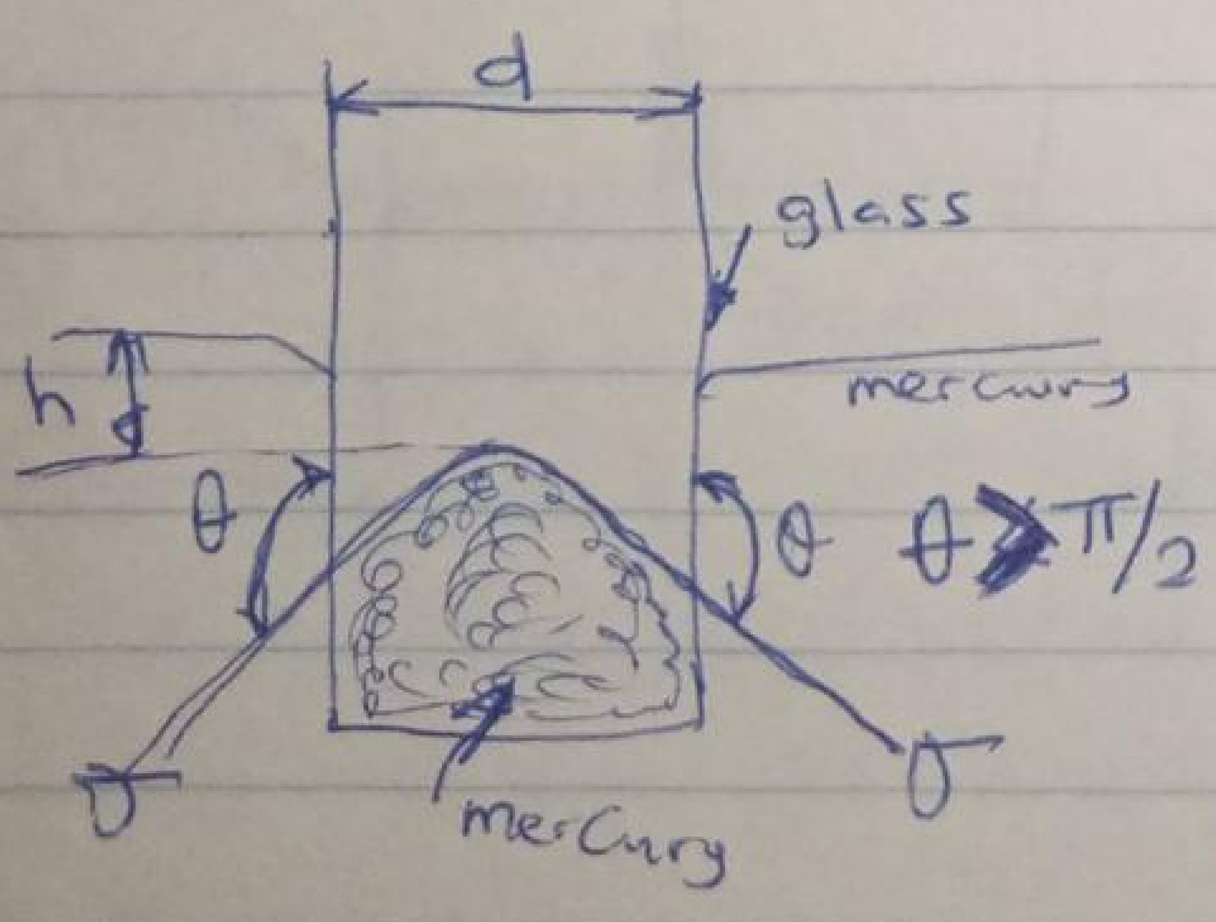
$$\pi d \cdot \sigma \cos\theta = \frac{\pi}{4} d^2 \times h \times W$$

$$\therefore h = \frac{4\sigma \cos\theta}{Wd}$$

for water and glass $\theta = 0$

$\therefore$ Capillary rise of water in the glass tube is $h = \frac{4\sigma}{Wd}$

For mercury there is a capillary depression as shown below with the angle of depression $\theta = 140^\circ$ $(\cos\theta = \cos 140^\circ)$ ~~=cos~~ it is equal to $-\cos 40$ hence h is negative



h = capillary depression

EXAMPLE — A clean tube of diameter 2.5 mm is immersed in a liquid with a Co-efficient of Surface tension 0.4 N/m. The angle of Contact of the liquid with the glass is 135°, The density of the liquid is 13600 Kg/m3.

Calculate the level of the liquid in the tube relative to the free surface of the liquid inside the Tube

Solution

Given $d = 2.5mm$, $\sigma = 4 N/m$, $\theta = 135°$, $\rho = 13600 kg/m^3$

$h = ?$

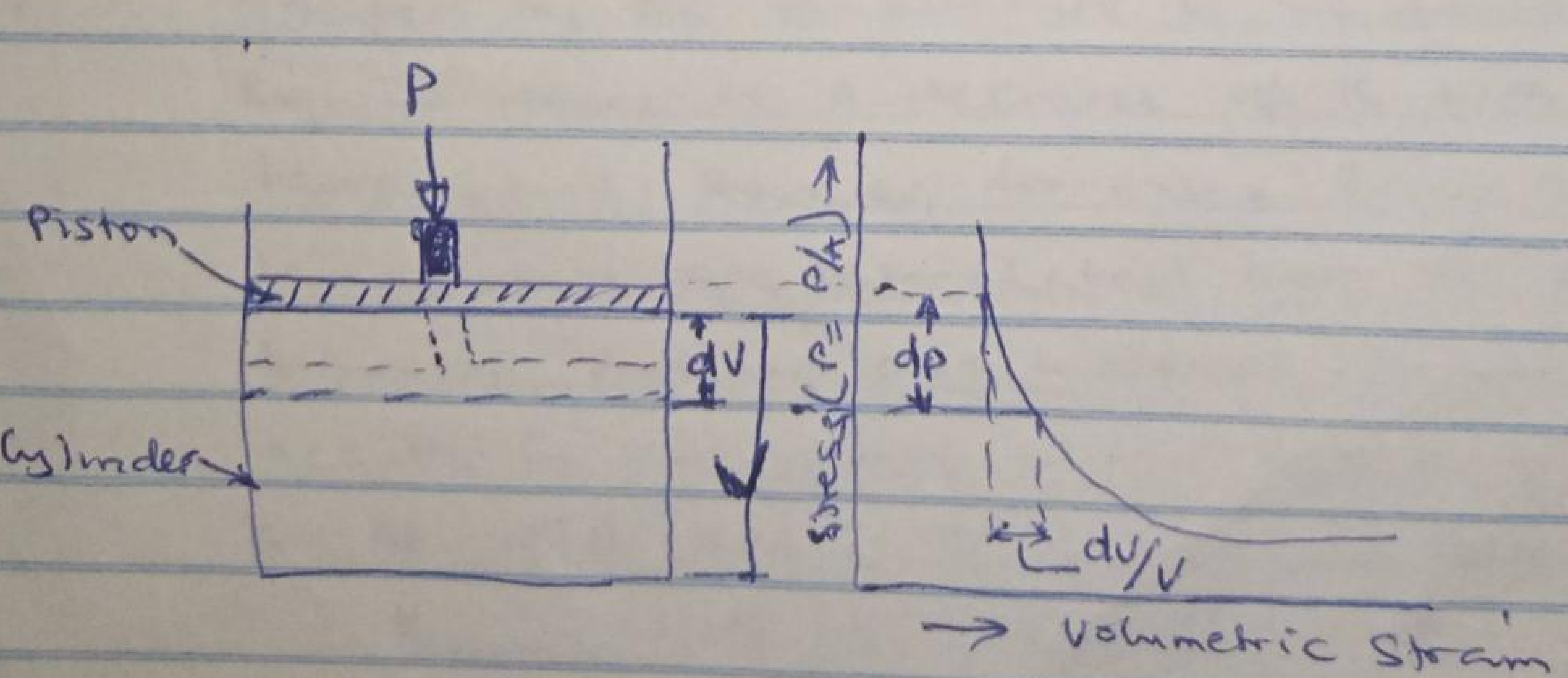
$$h = \frac{4\sigma \cos\theta}{wd}$$

$$= \frac{4 \times 0.4 \times \cos 135}{(9.81 \times 13600) \times 2.5 \times 10^{-3}}$$ (Note $w = \rho g$)

$= -3.39 \times 10^{-3}$ m or -3.39 mm This indicate depression

COMPRESSIBILITY AND BULK MODULUS

The Compressibility of a fluid is defined as a measure of change in volume of the fluid when it is subjected to the outside forces. The compressibility of any substance is a measure ~~of~~ in terms of bulk modulus of elasticity K. which is defined as the ratio of compressive stress to volumetric strain. ~~Compr~~ Compressibility is a reciprocal of bulk modulus of elasticity



V = volume of gas enclosed in cylinder
P = Pressure of gas when the volume is V
Let the pressure increased to P + dP, the volume of gas decreases from V to V − dV,
The increase in pressure = dP, decrease in volume = dV
∴ Volumetric strain = $-dV/V$ (the negative sign shows ~~that~~ there is decrease in volume with increasing pressure)
∴ bulk modulus K = $\frac{dP \text{ (increase pressure)}}{-dV/V \text{ (volumetric strain)}}$

i.e $K = \frac{dP}{-dV/V}$

(Compressibility = $1/K$)

The steepening of the curve in the diagram shows that with increasing pressure and compression of the fluid it is becoming increasingly to compress further. Which means the value K increases with increasing pressure.

Note this points •

1. The bulk modulus of elasticity (K) of a fluid is not constant, but increases with increase in pressure. This is because when a fluid mass is compressed its molecules becomes closer together and its resistance for further compression decreases i.e K increases

2. The bulk modulus of elasticity (K) of the fluid is affected by the temperature of the fluid. In the case of liquids there is a decrease of K with increase of temperature. However, for gases since pressure and temperature are interrelated and as the temperature increases, pressure also increases, an increase in temperature results in an increase in the value of K.

3. At NTP (Normal Temperature and Pressure)
$K_{water} = 2.07 \times 10^6$ kN/m² $K_{air} = 101.3$ kN/m²

Question — When a Pressure of 20.7 MN/m^2 is applied to 100 litres of a liquid its volume decreases by 1 litre. Find the bulk modulus of the liquid and Identify this liquid

Solution

Given, $dP = 20.7\ MN/m^2$, $dV = 1$ litre, $V = 100$ litres

$$\therefore -\frac{dV}{V} = \frac{1}{100}$$

$$K = -\frac{dP}{-dV/V} = \frac{20.7\times10^6}{1/100} = 20.7\times10^8\ N/m^2$$
$$= 2.07\ GN/m^2$$

QUESTION — Find an expression for Isothermal bulk modulus of elasticity for a gas which obey's van der waals law of state according to the equation

$$P = \rho RT\left[\frac{1}{1-b\rho} - \frac{a\rho}{RT}\right]$$

where a, b are Constants and ρ, R, and T have their usual meaning

Solution.

Bulk modulus of elasticity $K = \frac{-dP}{dV/V} = -V\frac{dP}{dV}$

Where, V is volume $= -v\,dP/dv$, v is define as specific volume

Since $v = 1/\rho$ or $\rho v = 1$, $dv/v = -d\rho/\rho$

$\therefore\ K = \cancel{\rho\,dP}\ \rho\frac{dP}{d\rho}$; $P = \rho RT\left[\frac{1}{1+b\rho} - \frac{a\rho}{RT}\right]$ given

$$\therefore \frac{dP}{d\rho} = RT\left[\frac{1}{1-b\rho} - \frac{a\rho}{RT}\right] + \rho RT\left[\frac{b}{(1-b\rho)^2} - \frac{a}{RT}\right]$$

$$= \frac{RT}{1-b\rho} - a\rho + \frac{b\rho RT}{(1-b\rho)^2} - a\rho$$

$$= \frac{RT}{1-b\rho}\left[1 + \frac{b\rho}{1-b\rho}\right] - 2a\rho = \frac{RT}{(1-b\rho)^2} - 2a\rho$$

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$$K = \rho \frac{dP}{d\rho} = \frac{\rho RT}{(1-b\rho)^2} - 2a\rho^2$$

Compression and expansion of gases takes place in accordance to various laws of thermodynamics.

- Isothermal process is a constant temperature process and is characterized by Boyle's law

PV = constant

$P_1V_1 = P_2V_2 = P_nV_n$

The bulk modulus can be derived by definition

$$K = -V\left(\frac{dP}{dV}\right) \quad ---(1)$$

and $PV = C \quad ---(2)$

$\therefore P = CV^{-1} \quad --(3)$

$dP/dV = -\frac{C}{V^2} \quad --(4)$

Substituting (4) in (1)

$K = -V(-CV^{-2}) = CV^{-1} \quad -(5)$

Substituting (2) in (5)

$K = (PV)V^{-1}$ or $\underline{K = P}$

Hence, in for an isothermal process, the pressure and the bulk modulus of elasticity are the same.

Isentropic process: Isentropic or reversible adiabatic process is a frictionless process in which no heat exchange across the boundary. This follows the relation

PV^{γ} = constant

where γ is adiabatic index, which is the ratio of two specific heats that is $\gamma = C_P/C_V$.

The bulk modulus can be derived as follows by definition