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Name: CVE

Class: 311

Subject: Fluid Mechanics II

School: CVE 312

Surface & Ground Water Hydrology

60 LEAV

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CVE 311: Fluid Mechanics

Course outline

Fluid Statics

Dynamics of Fluid flow

Conservation Equation of Mass and Momentum

Euler and Bernoulli's Equations

Introduction to Incompressible ~~fluids~~ viscous flow

Reynold's number, Dimensional analysis

Buckingham π-method, Rayleigh's method

Applications of Hydraulic

Flow models

Errors in measurement

Fluid statics

- Buoyancy
- Upthrust forces
- Pressure measuring, barometer

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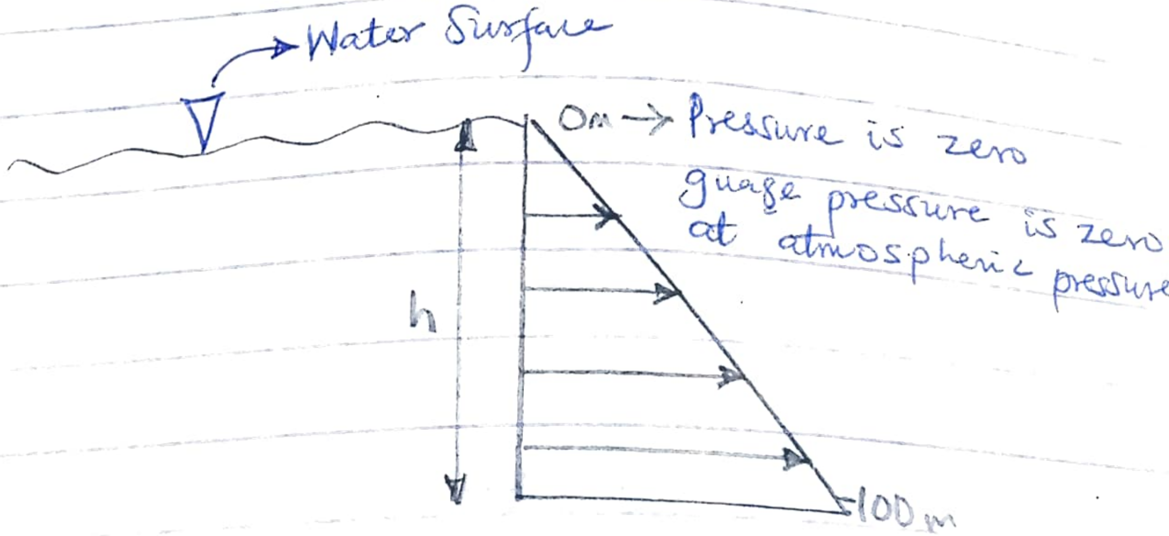
FLUID STATICS [HYDROSTATICS].

Fluids at rest have forces at perpendicular angles to the fluid.

Hydrostatics deals with the study of fluids at rest with focus on forces acting on the surrounding. The pressure (P) defined as the force exerted on a unit area of fluid, has it's unit expressed in Newton per metre square (N/m^2) or in Bars. 1bar = $10^5\ N/m^2$

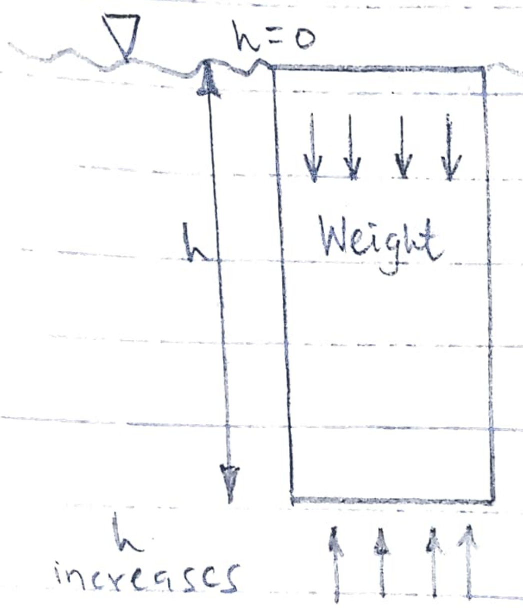
NOTE - Pressure in fluids at rest increases with increase in the depth of fluid.



As depth increases, pressure increases

For fluids at rest, all forces acting on a body of fluid are in equilibrium, i.e action and reaction forces are perpendicular to boundary surfaces

For fluids in motion, force within body of fluid exhibits a shear component therefore forces are not all perpendicular to boundary surfaces, the only force supporting a column of fluid at rest is the force acting upwards as shown below:



* Guage pressure at h=0
* Guage pressure = 0
* Weight acting downward
* ~~upward force~~

Fig1: Column of water with depth (h).

Weight of the column of water acting downward must be equal to the upward force.

The Specific weight of water = $\rho \times g$

$\rho \times g$

Density x Acceleration due to gravity.

$\therefore$ Therefore the weight of the column of water acting downward must be equal to the upward force

Weight of column of water

= Specific weight x volume

$= \rho g \times Ah$

Where ρ = density of water

g = Acceleration due to gravity

A = Cross-sectional area of the column of water.

h = Depth of column of water.

Always define your terms for exams before you end any calculation.

Upward force = Pressure (P) x Area

$PA = \rho g A h$

upward force = Weight acting downward

$P\cancel{A} = \rho g \cancel{A} h$

$\boxed{P = \rho g h}$ $\Rightarrow$ The Basic Hydrostatic Equation.

Guage Pressure (P)

(P) is the pressure with respect to atmospheric pressure. Guage pressure will always be zero at atmospheric pressure, Therefore Absolute pressure

$$P_{Abs} = P_{Atm} + \rho g h$$

Negative pressure

This is the pressure below atmospheric, which is called vacuum pressure

When using guage pressure for calculation atmospheric pressure is taken as datum, the equation "$\rho g h$" can be arranged for us to have

$$h = \frac{P}{\rho g}$$ ⇒ This is called the pressure head

Points of Equal Pressure in Hydrostatic Fluid

For hydrostatic fluid with a free water surface the hydrostatic pressure varies with increase in depth. All points along horizontal surface of fluid at rest are subjected to the same hydrostatic pressure. It must however be noted that for equal pressure to exist along a horizontal surface of fluid at rest

(1) The point considered in the surface must lie along/in the same liquid.

(2) The point must be continuous along the same elevation

(3) Continuity must exist within the fluid under consideration.

Measurement of Pressure

(1) Piezometer. This is a body of ~~fluid~~/liquid sealed under pressure. It is pierced with a vertical transparent tube called a piezometer. The liquid in the tube will

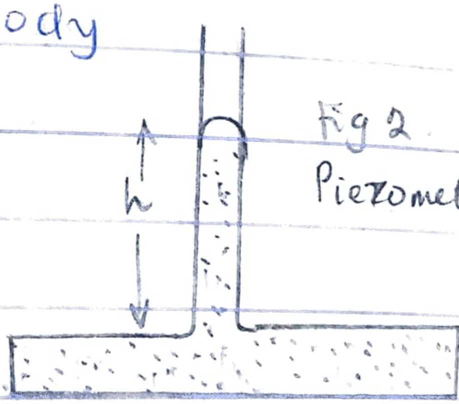


Fig 2 Piezometer

rise with a height (h) or to a height (h)
and is defined as

$$h_w = \frac{P}{\rho_w g} \quad \left[\text{Pressure head}\right].$$

The use of a piezometer is limited, as the height of the water column in the tube becomes unreasonably large for large pressure measurement. The use of a piezometer in measuring pressure in gas pipeline is unpracticeable. A range of pressure~~s~~ measuring devices have therefore been developed.

(2) Manometer

The manometer is a U-Shaped transparent tube. The fluid in the manometer is usually different from the fluid of which the pressure is to be measured. This guage fluid has a different density from the fluid to be measured. This guage fluid has a different density.

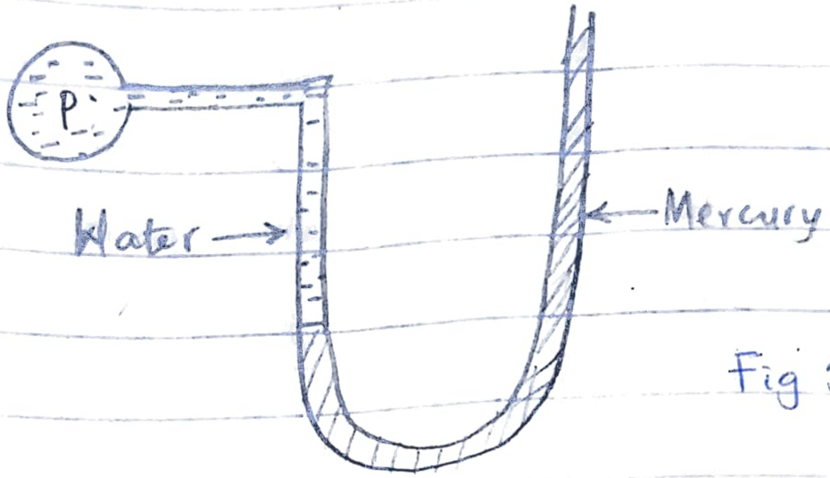
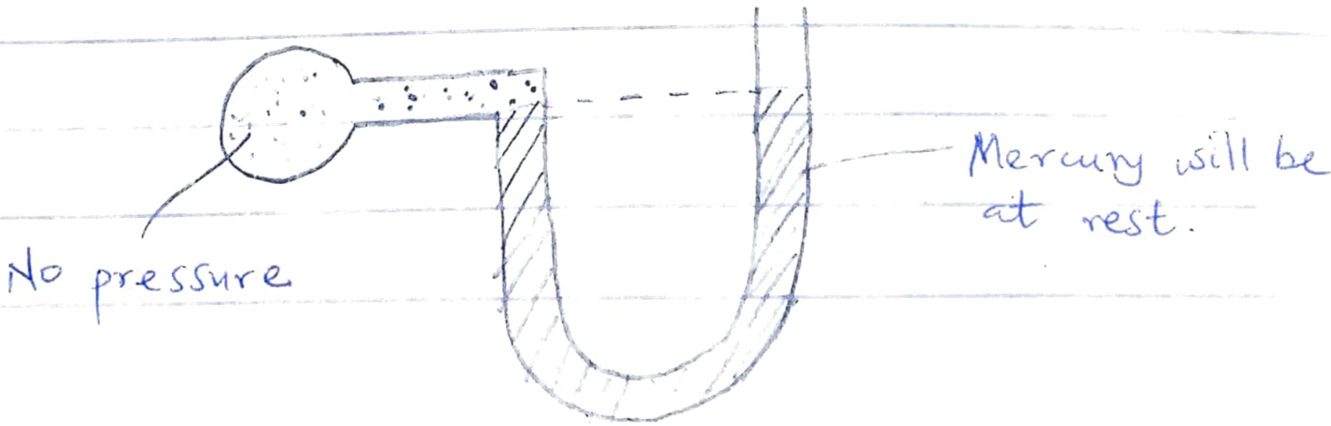


Fig 3 - Manometer

Mercury is usually used as the guage fluid. It is immiscible with water and does not react chemically.

If the guage pressure in the body of fluid to be measured is zero (0), the surface of the guage fluid in the left hand of the manometer will be at the same horizontal level with the open surface of the manometer fluid at the right hand of the manometer.



Example 1

Show that the pressure head in a body of fluid (water) with pressure (P) measured using a mercury manometer is given by :

$$\frac{P}{\rho_w g} = \frac{\rho_m}{\rho_w} h_2 - h_1$$

Solution Define all terms

P = Pressure in the body of fluid

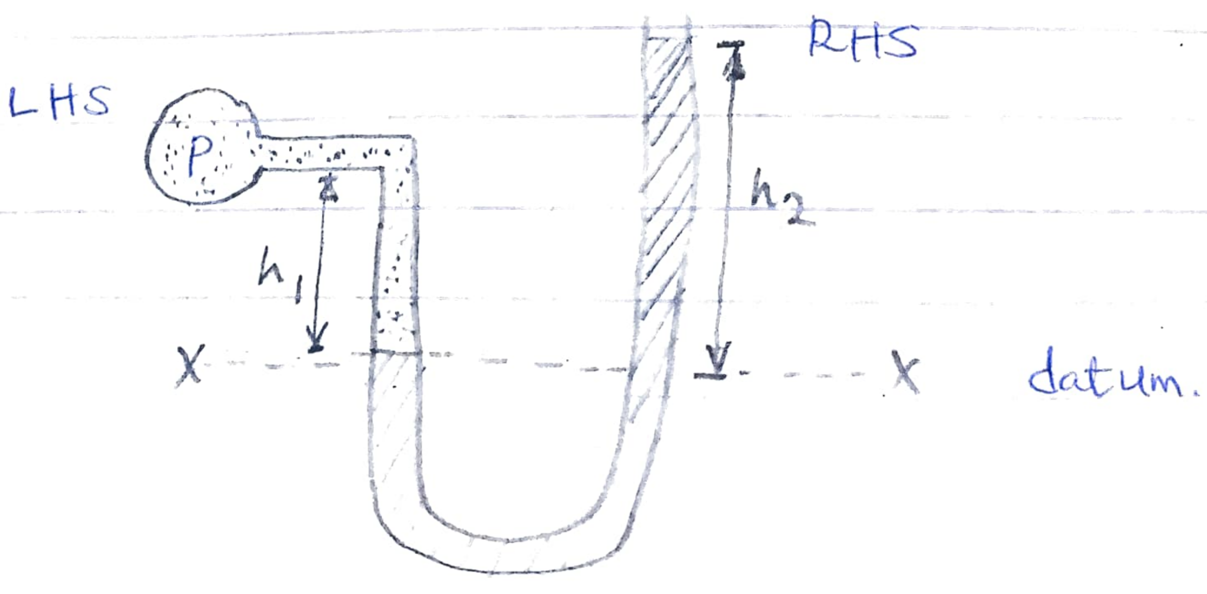
ρ_w = Density of water

g = Acceleration due to gravity

ρ_m = Density of mercury

h_2 = Height of column of mercury in the right limb of the manometer

h_1 = Height of column of water in the left limb of the manometer.



The horizontal datum X - X is set at the interface of water and mercury level, the total pressure on the left side of the manometer must be equal to the total pressure on the right side of the manometer from the set datum. On the left side of the manometer, total pressure is given by pressure at the centre of the body of water + Pressure due to the height h_1 for the water in the limb.

At the LHS, $P_{x-x} = P + \rho g h$

i.e $P_{x-x} = P + \rho_w g h_1$ —— ①

At the RHS, $P_{x-x} = \rho_m g h_2$ —— ②

Equate ① and ②

LHS = RHS "at the point X - X"

$P + \rho_w g h_1 = \rho_m g h_2$

$P = \rho_m g h_2 - \rho_w g h_1$

Divide through by g

$$\frac{P}{g} = \frac{\rho_m \cancel{g} h_2}{\cancel{g}} - \frac{\rho_w \cancel{g} h_1}{\cancel{g}}$$

$$\frac{P}{g} = \rho_m h_2 - \rho_w h_1$$

Divide through by ρ_w

$$\frac{P}{g\rho_w} = \frac{\rho_m h_2}{\rho_w} - \frac{\cancel{\rho_w} h_1}{\cancel{\rho_w}}$$

$$\therefore \boxed{\frac{P}{\rho_w g} = \frac{\rho_m}{\rho_w} h_2 - h_1}$$

Assignment 1

Dams are good examples of water bodies exhibiting hydrostatic pressure

Present a detailed writeup of 3 dams in Nigeria giving

(1) Name and date of construction

(2) Location of Dams

(3) Dimension of Dam

(4) Uses of Dam.

List of all dams/reservoirs in Nigeria with their location

Kainji dam (Niger state)
Jebba dam (Niger state)
Shiroro dam (Niger state)
Asejire Reservoir (Oyo State)
Bakolori dam (Sokoto state)
Challawa Gorge Dam (Kano state)
Dadin Kowa dam (Gombe state)
Goronyo dam (Sokoto state)
Gusau dam (Zamfara state)
Ikere Gorge dam (Oyo state)
Jibiya dam (Katsina state)
Katim dam (proposed) (Bauchi State)
Kiri dam (Adamawa state)
Mambilla dam (Taraba State)
Obudu dam (cross-river state)
Oyan River dam (Ogun state)

Tiga dam (Kano state)
Zauro polder project (Kebbi state)
Zobe dam (Katsina state)

\

$M \; L \; T \; \theta$

Velocity - LT^{-1}

Acceleration - LT^{-2}

Pressure or Stress · $ML^{-1}T^{-2}$

Angular velocity T^{-1}

Energy, Heat, Work $ML^{2}T^{-2}$

Power $ML^{2}T^{-3}$

Density · ML^{-3}

Viscosity . $ML^{-1}T^{-1}$

Sp. heat $L^{2}T^{-2}\theta^{-1}$

Discharge $L^{3}T^{-1}$

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Hydraulic Similitude and Model Analysis

Although many problems in engineering can be solved through equations and analytical procedures, a large number of problems require experimental data to solve them. The solution is obtained by the use of a combination of analysis and experimental data.

Similitude is used here, so that measurements made on one system can be transferred to another

Dimensional Analysis.

This is used to predict the physical parameters that will significantly influence a phenomenon under study. This dimensional analysis uses MLT - The basic analysis.

M - Mass

L - Length

T - Time.

There are other secondary (derived) units e.g

Pressure - N/m^2

$$\frac{\text{mass} \times \text{Acceleration}}{m^2} = MLT^{-2}$$

All derived formulas have their basis from MLT.
The principal analysis is called the MLT Analysis.

The flow characteristics, Unit & Dimension.

- Geometric Characteristics
- Kinematic Characteristics
- Dynamic characteristics.

① Geometric Characteristics

Length, Area, Volume. (analysis)

m (L) m^2 (L^2) m^3 (L^3)

② Kinematic Characteristics

Time, Velocity Acceleration, Discharge

s (T) m/s LT^{-1} m/s² (LT^{-2}) m³/s = L^3T^{-1}

③ Dynamic Characteristics

Mass, Force Pressure, Energy, Power

kg (M)

Force - N - kg m/s² (MLT^{-2})

Pressure - $\frac{\text{Force}}{\text{Area}}$ - $\frac{N}{m^2}$ $\frac{MLT^{-2}}{L^{-2}}$ $\Rightarrow ML^{-1}T^{-2}$

Energy - Joules Nm $\quad MLT^{-2} \cdot L \Rightarrow (ML^2T^{-2})$ ✓

Power - Watts Nms^{-1} $\quad \frac{ML^2T^{-2}}{T} = (ML^2T^{-3})$ ✓

~~Methodology~~

<u>Methods of Dimensional Analysis.</u>

The basic principle is homogeneity - Dimensions on both sides of the equation must be the same. e.g $V = (2gH)^{1/2}$

V = Velocity

g = Acc. due to gravity

H = Height ~~(time)~~

$$V = (2gH)^{1/2}$$

$$LT^{-1} = (2 \cdot LT^{-2} \cdot L)^{1/2}$$

$$LT^{-1} = L^{1/2}\,T^{-1} \cdot L^{1/2}$$

$$LT^{-1} = L^{1/2}\,T^{-1/2}$$

$$LT^{-1} = LT^{-1}$$

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Methods of Dimensional Analysis

1. Rayleigh's Method.

This is used to determine expressions for a variable (dependent variable) which depends on a maximum of 3-4 variables (independent variables). It does not exceed 3-4 independent variables.

Example 1

The resulting force R of a supersonic plane during flight can be considered as dependent on the length L of the aircraft, velocity V, air viscosity μ, air density ρ, and bulk modulus of air K. Express the fundamental relationship between the variables and the resulting forces.

Solution

$$R = f(L, V, \mu, \rho, K)$$

Introduce a non-dimensional alphabet (constant) A.

$$\therefore R = A(L, V, \mu \rho, K)$$

R = Force = N (kg m/s²) MLT^{-2}

Length = L (m) = L

V = m/s = LT^{-1}

$\mu = ML^{-1}T^{-1}$

ρ = kg/m³ = ML^{-3}

$K = ML^{-1}T^{-2}$

Length V   Velocity m/s   Viscosity   Density kg m⁻³   Bulk modulus

$$MLT^{-2} = A(L)(LT^{-1})(ML^{-1}T^{-1})(ML^{-3})(ML^{-1}T^{-2})$$

Introduce powers a, b c . . . on each of the dependent variables.

$$MLT^{-2} = A[L]^a[LT^{-1}]^b[ML^{-1}T^{-1}]^c[ML^{-3}]^d[ML^{-1}T^{-2}]^e$$

Equating the variables,

M: $1 = c + d + e$ —— (1)

L: $1 = a + b - c - 3d - e$ —— (2)

T: $-2 = -b - c - 2e$ —— (3)

From equation (1),

$d = 1 - c - e$

Express a, b and c in terms of d and e.

From (3),

$b = 2 - c - 2e$

From (2), $a = 1 - b + c + 3d + e$

$$a = 1-(2-c-2e)+c+3(1-c-e)+e$$

$$a = 1-2+c+\cancel{2e}+c+3-3c-\cancel{3e}+\cancel{e}$$

$$a = 2-c$$

e has exceeded 4

$$R = AL^a V^b M^c \rho^d K^e$$

$$R = AL^{2-c} V^{2-c-2e} M^c \rho^{1-c-e} K^e$$

$$R = AL^2 V^2 \rho^1 (L^{-c} V^{-c} M^c \rho^{-c})(V^{-2e} \rho^{-e} K^e)$$

$$R = A\rho L^2 V^2 \left(\frac{M}{LV\rho}\right)^c \left(\frac{K}{V^2 \rho}\right)^e$$

* The velocity of propagation of pressure wave through a liquid can be expected to depend on the elasticity of the liquid represented by the bulk modulus K and it's mass density ρ. Establish by dimensional analysis, the relationship forms possible.

$A = -b$

$B = 1-2d$

$C = 3+3a+b-d$

choice of repeating variables

$d V \rho$
$L V \rho$
$d V M$

2. Buckingham's π - Theorem

Since Rayleigh's method becomes laborious when variables are more than the fundamental dimensions (MLT), the difficulty is overcome by buckingham's π - theorem, which states that If there are n-variables, independent and dependent in a physical phenomenon, and contains m-fundamental dimensions then the variables are arranged in (n-m) terms, otherwis known as π terms

Rules

1. Each π term is dimensionless and is independent of systems of unit
2. Division or multiplication by a constant does not change the character of the π-terms
3. Each π-term contains m+1 variables, where m is the number of fundamental dimensions, and is also called repeating variable

Example

A thin rectangular plate having a width w and height h is located so that is normal to a moving stream of fluid. Assume the drag force d that the fluid exerts on the plate is a function of w and h, ~~the~~ fluid viscosity μ and density ρ respectively and velocity V of the fluid approaching the plate. Determine a suitable set of π-terms to study this problem experimentally

Solution

$D = f(W, h, \mu, \rho, V)$ $W h \mu$ (ρV)

Total no of variables $= 6 = n$

no of fundamental dimensions $m = 3$

$(n-m) =$ no of π-terms

$= (6-3) = 3\pi$-terms

Repeating variables

$w \rho V$

$D = MLT^{-2}$

$h = L$ $\rho = ML^{-3}$

$W = L$

$\mu = ML^{-1}T^{-1}$

$V = LT^{-1}$

π - terms available

$\pi_1 = D, W^a, \rho^b V^c$

$\pi_2 = h W^a \rho^b V^c$

$\pi_3 = \mu W \rho V$

$\pi_1 = M^0 L^0 T^0 = (MLT^{-2})(L)^a (ML^{-3})^b (LT^{-1})^c$

compare variables

$M: \quad 0 = 1 + b \quad — (1)$

$L: \quad 0 = 1 + a - 3b + c \quad — (2)$

$T: \quad 0 = -2 - c \quad — (3)$

from (1), $b = -1$

from (3), $c = -2$

from (2), $0 = 1 + a - 3b + c$

$a = 3b - c - 1$

$a = 3(-1) - (-2) - 1$

$a = -3 + 2 - 1$

$a = -2$

$a = -2$

$b = -1$

$c = -2$

$\pi_1 = D W^{-2} \rho^{-1} V^{-2}$

$$\pi_1 = \frac{D}{W^2 \rho V^2}$$

π_2

$$M^0 L^0 T^0 = (L)(L)^a (ML^{-3})^b (LT^{-1})^c$$

compare variables

M: $0 = b$ — ①

L: $0 = 1 + a - 3b + c$ — ②

T: $0 = -c$ — ③

from ①, $b = 0$,

from ③, $c = 0$

from ②, $a = 3b - 1 - c$

$a = 3(0) - 1 - 0$

$a = -1$

$a = -1$
$b = 0$
$c = 0$

$\pi_2 = h\, w^{-1} \rho^0 V^0$

$\pi_2 = \frac{h}{w}$

π_3

$$M^0 L^0 T^0 = ML^{-1}T^{-1} (L)^a (ML^{-3})^b (LT^{-1})^c$$

M: $0 = 1 + b$ — ①

L: $0 = -1 + a - 3b + c$ — ②

T: $0 = -1 - c$ — ③

$b = -1$

$c = -1$

$a = 3b + 1 - c$

$$a = 3(-1) + 1 - (-1)$$
$$a = -3 + 1 + 1$$
$$a = -3 + 2 = -1$$

$a = -1$
$b = -1$
$c = -1$

$$\pi_3 = \mu\, w^{-1} \rho^{-1} V^{-1}$$
$$\pi_3 = \frac{\mu}{w \rho V}$$

$$\pi_1 = \phi(\pi_2, \pi_3)$$

π_1 is a function of π_2 and π_3

ϕ = dimensionless model.

* ✓ Assignment

The Discharge through a ~~well~~ wier is $1.5\ m^3/s$, Find the discharge through the model of well if the horizontal dimensions of the model $= \frac{1}{50}$, the horizontal dimensions of prototype, and vertical dimension of model $= \frac{1}{10}$ of vertical dimension of prototype

$$\frac{Q_P}{Q_m} = (L_r)_H \, (L_r)_V^{3/2}$$

$$\frac{Q_P}{Q_m} = \frac{L_P^2 V_P}{L_m^2 V_m}$$

$$\boxed{\begin{aligned}(L_r)_H &= \frac{L_P}{L_m} = 50 \\ (L_r)_V &= \frac{L_P}{L_m} = 10\end{aligned}}$$

$$\frac{Q_P}{Q_m} = 50 \times (10)^{3/2}$$

$$\frac{1.5}{Q_m} = 50 \times 10^{3/2}$$

$$Q_m =$$

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Model Studies and Similitude

Similitude or dynamic similarities between two similar systems exist when the ratio of the initial force in the individual force components in the first system is the same as the corresponding point in space

For absolute dynamic similarities, we need

$$\left.\begin{array}{l} \text{Reynold's number} \\ \text{Froude's number} \\ \text{Webber's number} \end{array}\right\} \text{Dynamic similarities.}$$

Prototype — bigger
Model — smaller

Geometric Similarities

The model must be the same shape as the prototype.

Laboratory scale - ~~Prototype~~ Model

$$\frac{L_p}{L_m} = \frac{D_p}{D_m} = \frac{H_p}{H_m} = \text{Length ratio } (L_r)$$

$$\frac{A_p}{A_m} = \frac{L_p \cdot D_p}{L_m \cdot D_m} = (L_r)^2$$

$$\frac{V_p}{V_m} = \frac{L_p \cdot D_p \cdot H_p}{L_m \cdot D_m \cdot H_m} = (L_r)^3$$

All 3 conditions for similarity must occur before similitude/similarity exists.

The velocity at a [illegible] point in the model [illegible] must be proport[illegible] velocity of [illegible] time-scale equivalent

Kinematic Similarities

Exist between models and prototypes - (Time and velocity)

$$\frac{Vp_1}{Vm_1} = \frac{Vp_2}{Vm_2} = Vr$$ (velocity)

$$\frac{Ap_1}{Am_1} = \frac{Ap_2}{Am_2}$$

$$\frac{Am_1}{Am_2} = \frac{Ap_2}{Ap_2} = Ar$$ (acceleration)

Dynamic Similarities - Both geometric and kinematic

$$\frac{Fp_i}{Fm_i} = \frac{Fp_v}{Fm_v} = \frac{Fp_g}{Fm_g} = Fr$$

(inertia force, viscous force, gravity)

Inertia force = mass X Acceleration (flowing fluid)

Viscous force = Shear Stress (viscosity) X Surface area

Gravitational force = mass X Acceleration due to gravity.

Surface Tension force F_s = Surface Tension X Length of Surface

Dimensionless numbers - Numbers gotten by dividing the Inertia force by

- viscous force
- gravity force
- pressure force
- Surface Tension
- Elastic force

$$\frac{\text{Inertia force}}{\text{Viscous force/gravity/e.t.c}}$$

Reynold's

Euler

Froude

Reynold's Number = $\frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v} = \frac{\text{mass} \times \text{acceleration}}{\text{Shear Stress} \times \text{Area}}$

$$= \frac{\text{mass} \times \frac{\text{Velocity}}{\text{Time}}}{\text{Shear Stress} \times \text{Area}}$$

$$= \frac{\rho\ \text{volume} \times \frac{\text{Velocity}}{\text{Time}}}{\text{Shear Stress} \times \text{Area}}$$

$$\frac{\rho\ \frac{\text{volume}}{\text{Time}} \times \text{Velocity}}{S.S \times \text{Area}}$$

$$\frac{\rho Q V}{S.S \times \text{Area}}$$

$$\frac{\rho A V \cdot V}{S.S \times \text{Area}}$$

$$\frac{\rho A V \cdot V}{\tau \cdot A} = \frac{\rho A V \cdot V}{\mu \frac{du}{dy} \cdot A} = \frac{\rho A V \cdot V}{\mu \frac{V}{L} \cdot A}$$

$$= \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

$(\rho L^2 V^2)$. Force ratio

$$\frac{\text{Inertia force}}{\text{Gravitational force}} = \text{Froude's number } (Fe^2)$$

$$\frac{F_i}{F_g} = Fe^2$$

$$Fe^2 = \frac{F_i}{F_g} = \sqrt{\frac{F_i}{F_g}} = \sqrt{\frac{\text{mass} \cdot \frac{\text{velocity}}{\text{Time}}}{\text{mass} \times \text{Acceleration due to gravity}}}$$

$$Fe = \sqrt{\frac{\rho \cdot \frac{\text{vol}}{\text{Time}} \cdot \text{velocity}}{\text{Mass} \times \text{Acc.}}}$$

$$Fe = \sqrt{\frac{\rho Q V}{\rho \times \text{vol} \times g}}$$

$$Fe = \sqrt{\frac{\rho A V \cdot V}{\rho A L g}}$$

$$Fe = \sqrt{\frac{V^2}{gL}} \qquad \frac{\sqrt{V^2}}{\sqrt{gL}}$$

Euler's Number (E_u) - $\frac{\text{Inertia force}}{\text{Pressure force}}$ $\frac{F_i}{F_p}$

$$E_u^2 = \frac{F_i}{F_p}$$

$$E_u = \sqrt{\frac{F_i}{F_p}}$$

$$= \sqrt{\frac{\text{Mass} \cdot \frac{\text{Vel}}{\text{Time}}}{\text{Pressure} \times \text{Area}}}$$

$$= \sqrt{\frac{\rho \times \frac{\text{Vol}}{\text{Time}} \times \text{Velocity}}{\text{Pressure} \times \text{Area}}}$$

$$= \sqrt{\frac{\rho Q V}{PA}}$$

$$= \sqrt{\frac{\rho \cancel{A} V \cdot V}{P \cancel{A}}}$$

$$= \sqrt{\frac{\rho V^2}{P}}$$

* Webber's Number $= \dfrac{\text{Inertia Force}}{\text{Surface Tension}}$ $\qquad \dfrac{V}{\sqrt{\frac{\sigma}{\rho L}}}$

* Mach's Number $M = \dfrac{\text{Inertia Force}}{\text{Elastic force}}$ $\qquad \dfrac{V}{C}$

$C = \sqrt{K/\rho}$

Similarity Laws

The forces of inertia comes into play when all other forces, summed up, is not equal to zero

$$F_v + F_g + F_p + F_s + F_e = F_i$$

$$\frac{(F_v + F_g + F_p + F_s + F_e)_{prototype}}{(F_v + F_g + F_p + F_s + F_e)_{model}} = \frac{F_i\ (prototype)}{F_i\ (model)}$$

Example

Reynold's Similarity Law

* A pipe of diameter 1.5m is required to transport an oil of specific gravity 0.90 and viscosity 3×10^{-2} poise at a rate of 3000 L/s. Tests were conducted on a 15cm diameter pipe using water at 20°C. Find the velocity and flow rate in the model.

Solution

Prototype data

Diameter $D_p = 1.5m$

Viscosity of fluid $\mu_p = 3 \times 10^{-2}$ poise

Discharge $Q_p = 3000\ Ls^{-1}$ ($3m^3/s$)

Specific gravity $S_p = 0.9$

Density of oil $= \rho_p = \boxed{0.9 \times 1000}$

$= 900 kg/m^3$

Model data

Diameter of model $D_m = 15cm = 0.15m$

Viscosity μ $\mu_m = 1 \times 10^{-2}$ poise (Dynamic

$\rho_m = 1000 kg/m^3$ (density of water)

$V_m = ?$

3000 L/s ⇒ 3 m³/s

$Q_m = ?$

$$\frac{\rho_m \times V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$V_p = \frac{Q_p}{A_p}$ (from $Q = VA$).

$$\boxed{\begin{aligned} V_m &= \frac{Q_m}{A_m} \quad 5.09 \text{ m} \\ Q_m &= V_m A_m \quad 0.0899 \text{ m}^3/s \end{aligned}}$$

$$V_p = \frac{Q_p}{A_p} \qquad A_p = \frac{\pi d^2}{4} = \frac{\frac{22}{7} \times 1.5^2}{4}$$

$$= 1.767 m^2$$

$$V_p = \frac{3 m^3/s}{1.767 m^2}$$

$$V_p = 1.698 m/s$$

$$\frac{\rho_m \times V_m \times D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}$$

$$\frac{1000 \times V_m \times 0.15}{1 \times 10^{-2}} = \frac{900 \times 1.698 \times 1.5}{3 \times 10^{-2}}$$

$$V_m = \frac{1 \times 10^{-2} \times 900 \times 1.698 \times 1.5}{1000 \times 0.15 \times 3 \times 10^{-2}}$$

$$V_m = 5.092 \text{ m/s}$$

$$\frac{Q_m}{A_m} = V_m$$

$$Q_m = V_m A_m = 5.092 \times \frac{\pi \times 0.15^2}{4}$$

$$= \underline{0.0899 \text{ m}^3/\text{s}}$$

A ship 300 m long moves in sea water, whose density is 1030 kg/m³. A 1:100 model of this ship is to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30 m/s and the resistance of the model is 60 N. Determine the velocity of the ship in sea water, and also the resistance of the ship in sea water.

$\rho_{air} = 1.24 \text{ kg/m}^3$

$\mu_{air} = 0.012$ stokes

$\mu_{water} = 0.018$ stokes

Derivation of Euler & Bernoulli Equation of Motion

22/02/2021.

Euler and Bernoulli's Equation

The Euler equation for steady flow of an ideal ~~equation~~ fluid along a streamline is a relation between velocity, pressure and density of a moving fluid. It is based on Newton's 2nd law of motion, and the integration of Euler's equation gives Bernoulli's equation in the form of energy per unit ~~[illegible]~~ weight of the flowing fluid

Assumptions

1 - The fluid is non-viscous ~~[illegible]~~ i.e friction losses are zero

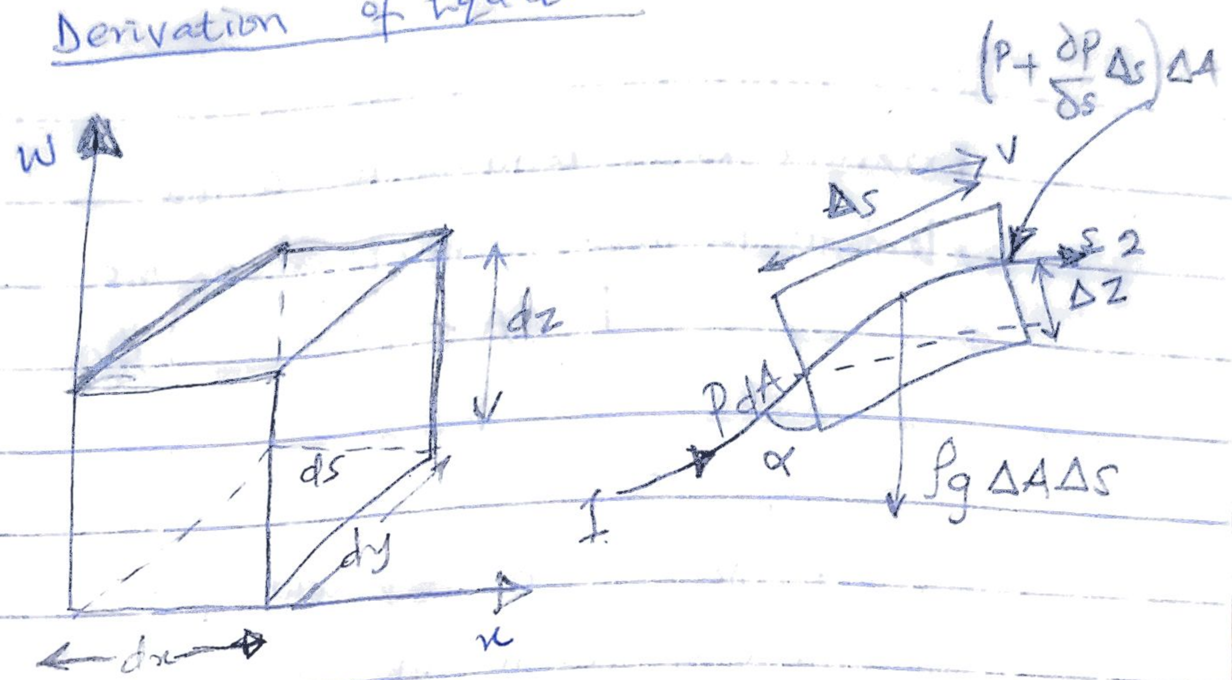
2 The fluid is homogenous and incompressible (mass and density of the fluid is constant

3 The flow is continuous and steady along the streamline

4. The velocity of the flow is uniform over the section.

5 No energy or force except gravity and pressure force is involved in the flow.

Derivation of Equation



$$\text{Pressure} = \frac{\text{force}}{\text{Area}}$$

force = Pressure x Area.

PdA = force at point 1

$\left(P+\frac{\delta P}{\delta s}\Delta s\right)\Delta A$ = force at point 2.

Take dA = cross-sectional area of the fluid element

ds = Length of fluid element

dW = Weight of the fluid element

P = pressure on the fluid element at point 1

$P+\delta P$ = pressure on the element at point 2.

V = velocity of the fluid element

$\frac{dp}{\rho} + g \cdot dz + v\,dv = 0$ Euler's eq.

External forces tending to accelerate the fluid element in the direction of the streamline

$= P \cdot dA - (P + \delta P)\,\delta A$

Initial force

$P\cancel{dA} - \cancel{PdA} - \delta P\,\delta A$

$= -\,\delta p \cdot \delta A$

$\delta W = \rho g\, \delta A \cdot \delta s$ (= Weight of the fluid element)

Component of the weight in the same direction of flows $= \rho g \cdot dA \cdot ds \cos\theta$

$= \rho g \cdot dA \cdot \cancel{ds}\, \frac{dz}{\cancel{ds}}$

$\rho g\, dA\, dz$ ——— (1)

Mass $= \rho\, dA \cdot ds$

Acceleration of fluid element

$\frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v\frac{dv}{ds}$ ——— (2)

From Newton's 2nd law, $F = ma$

$-dp \cdot dA - (\rho g \cdot dA \cdot dz) = \rho dA \cdot ds \times V \cdot \frac{dv}{ds}$

$\Rightarrow \boxed{\frac{dP}{\rho} + g \cdot dz = -v\,dv}$ [Euler's Equation]

Pressure head

$$\frac{P_1}{\rho g} + z_1 + \frac{V_1}{2g} = \frac{P_2}{\rho g} + Z_2 + \frac{V_2}{2g}$$ [Bernoulli's Equation]

$$\frac{P_1}{\rho g} = \text{Pressure head}$$

$$\frac{V_1}{2g} = \text{gravitational head}$$

There are 3 different cases.

(1) When the pipe nozzles are closed, the total head $H = \frac{P}{\rho g} + Z$ (velocity is zero).

(2) When pipe nozzles are opened, and flow is allowed, Total head $= \frac{P}{\rho g} + \frac{V^2}{2g} + Z$.

(3) Head loss computation:
They are generally energy losses due to friction in a real-world order; and must be taken into account when relating with fluids.

-lar conduit, there are different

diameter $d_1 = 2m$, which changes

3m. The velocity in the entrance profile

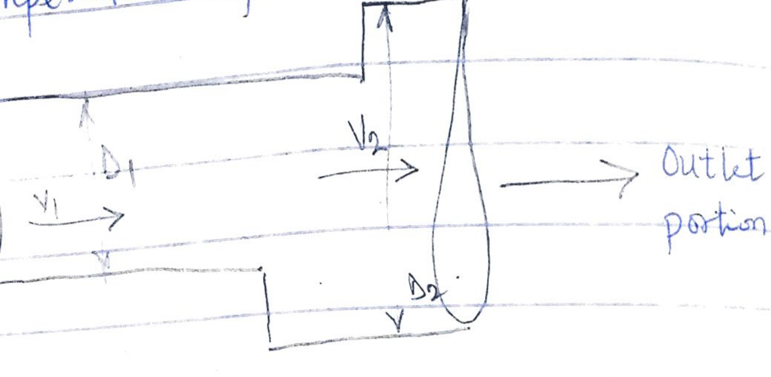
-d as $3ms^{-1}$. Calculate the discharge

velocity at the outlet profile.

also the type of flow in both

profiles, if it is laminar or turbulent

mperature of $12°c$.



the continuity equation, $Q = AV$ or $V_1 S_1$

$$Q = V \cdot \frac{\pi d^2}{4}$$

V_1 = Inlet velocity = $3ms^{-1}$

d_1 = diameter of inlet = $2m$

$$Q = \frac{3 \times \pi \times 2^2}{4}$$

$Q_1 = 9.425\ m^3/s$. ⇒ Inlet flow rate

P pgh
p
pg

$d_2 = 3m$

$Q_2 = A_2 V_2$

$V_2 = \frac{Q}{A_2}$

$= \frac{Q}{\frac{\pi \times 3^2}{4}} = 1.33 \, m/s.$

$Re = \frac{VD}{\mu}$

Laminar 2320 < Reynold's number (Re)
Turbulent 2320 > Reynold's no. (Re)
Ideal/Steady 2320 = Reynold's no (Re).

The kinematic viscosity of water @ 12°C = 1.24×10^{-6}

$\therefore Re_1 = \frac{V_1 D_1}{\mu}$

$= \frac{3 \, m/s \times 2}{1.24 \times 10^{-6}} = 4838709$

$Re_2 = \frac{V_2 D_2}{\mu} = \frac{1.33 \times 3}{1.24 \times 10^{-6}} = 3217741$

~~The flow is laminar~~

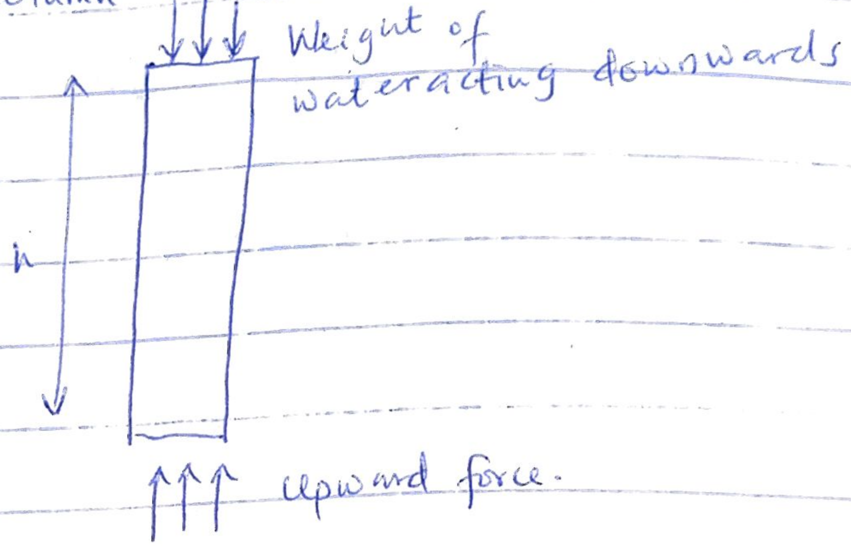
(2) A ~~reservoir~~ horizontal pipeline is attached to the wall of a reservoir. The pipeline has different profiles. The water level in the upper reservoir is in the height $h = 1.5m$ above the pipeline axis. From the lower end of the pipeline, water flows out to the open space. Diameters and length of pipeline ~~reaches~~ readings are $d_1 = 0.24m$, $L_1 = 3m$, $d_2 = 0.1m$ $L_2 = 1m$, $d_3 = 0.12m$, $L_3 = 2m$

Calculate the discharge in the pipeline and draw the course of energy line and pressure line.

REVISION NOTES

The basic hydrostatic equation $P = \rho g h$.

For a column of fluid at rest,



Upward forces = Downward forces

Downward force = Weight of the fluid = Specific weight X Volume.

where specific weight = ρg

ρ = density of fluid

g = Acceleration due to gravity.

Volume = Ah

A = Cross-sectional area of the column of water

h = depth of water column.

$\therefore$ Downward force = Specific Wt. X Volume

$= \rho g \times Ah$

$= \rho g A h$.

$P_{Abs} \; P_{gauge} + P_{atm}$

Upward force = Pressure × Area

$= PA$

Upward force = downward force

$PA = \rho g \cancel{A} h$

$\boxed{P = \rho g h}$ ⇒ The basic hydrostatic equation.

P = guage pressure

Absolute pressure = $P_{atm} + \rho g h$ — guage pressure.

Measurement of Pressure

- Piezometer

If a body of liquid sealed under pressure is pierced with a vertical transparent tube called a piezometer, the liquid will rise to a height h, which is defined as

$$h_w = \frac{P}{\rho g} \quad \text{(pressure head)}$$

$\rho g h$

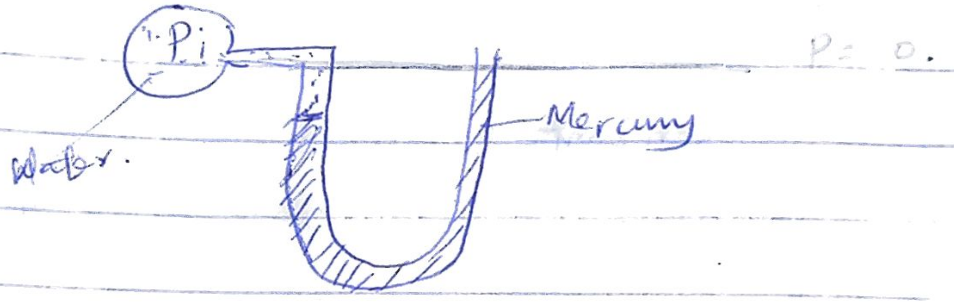
$\frac{P}{\rho g}$

- Manometer

The manometer is a U-shaped transparent tube. The fluid that is to be measured has a

- ~~ping~~ manometer
- The Bourdon gauge

different density than the gauge fluid in the manometer. Mercury is usually used as the gauge fluid of a manometer because it is immiscible with water, and does not react chemically

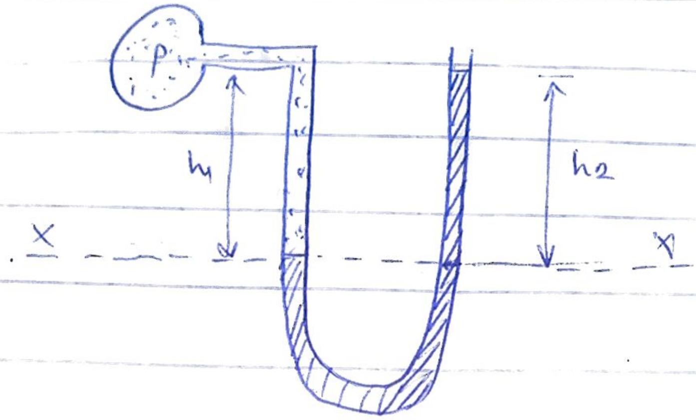


* Ex 1 - Show that the pressure head in a body of fluid with pressure P, measured using a mercury thermometer is given by

$$\frac{P}{\rho_w g} = \frac{\rho_m}{\rho_w} h_2 - h_1$$

- Identify all elements of the equation,
- draw diagram.

P = Pressure in the body of water
ρ_w = density of water
g = Acceleration due to gravity
ρ_m = density of mercury
h_2 = height of column of mercury on the right side of the manometer
h_1 = height of water column on the left side of the manometer.



A horizontal datum is set at the interface of the mercury and water level.

The pressure at the RHS of the manometer = Pressure at the LHS of the manometer.

At the LHS, the pressure = Pressure at the centre of water body + Pressure due to height h_1

$\therefore$ At LHS, Pressure = $P + \rho_w g h_1$ — (1)

At the RHS, pressure = pressure due to height h_2

$\therefore$ At RHS, pressure = $\rho_m g h_2$ — (2)

Equating (1) and (2)

$$P + \rho_w g h_1 = \rho_m g h_2$$

$$P = \rho_m g h_2 - \rho_w g h_1$$

ter

meter

ng manometer.

guage

Divide through by g

$$\frac{P}{g} = \frac{\rho_m g h_2}{\cancel{g}} - \frac{\rho_w g h_1}{\cancel{g}}$$

$$\frac{P}{g} = \rho_m h_2 - \rho_w h_1$$

Divide through by ρ_w

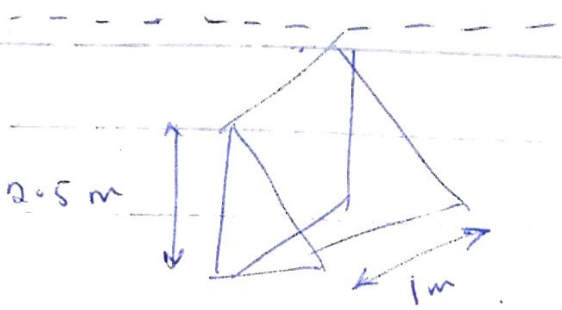
$$\frac{P}{g\rho_w} = \frac{\rho_m h_2}{\rho_w} - \frac{\cancel{\rho_w} h_1}{\cancel{\rho_w}}$$

$$\frac{P}{g\rho_w} = \frac{\rho_m h_2}{\rho_w} - h_1$$

$$\therefore \quad \frac{P}{\rho_w g} = \frac{\rho_m}{\rho_w} h_2 - h_1$$

Hydrostatic pressure on plane surfaces immersed in a body of fluid.

For a plate vertically suspended in water, with the top edge at the surface of the water



(1) Pressure at the surface = 0

(2) Pressure at the bottom (depth 2.5m)

$P = \rho g h$

$P = 1000 \times 9.81 \times 2.5 = 24{,}525\ N/m^2$

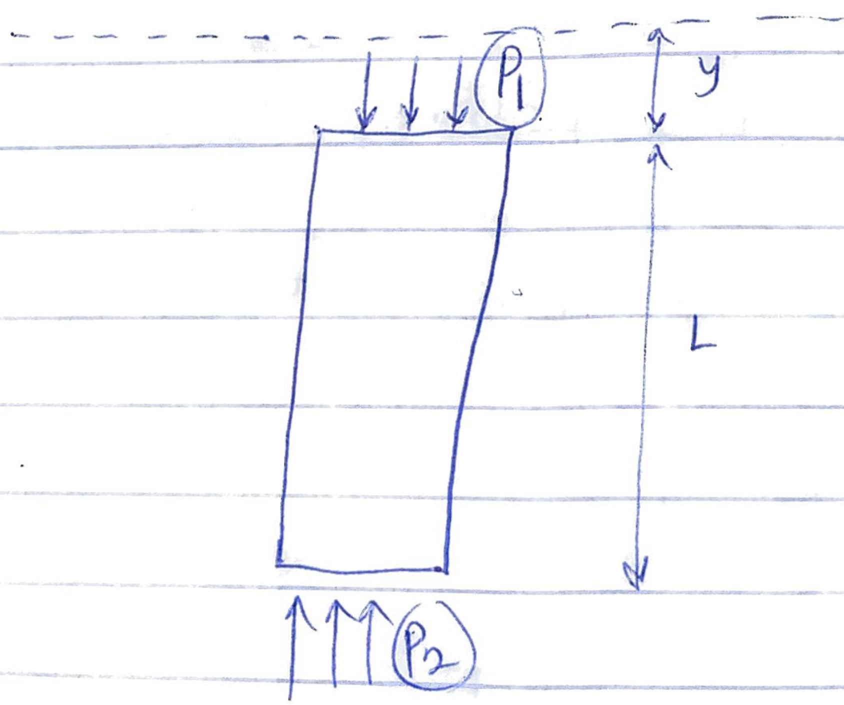
(3) Force on one side of the plate

$F = \text{Pressure} \times \text{Area}$

~~= 24525 ×~~

Buoyancy Forces

Consider a cylinder submerged in a body of fluid (water) with its vertical axis shown



Forces acting at the surface ——— (1)

$$P_1 A = \rho g y A$$

Force acting at the bottom ——— (2)

$$P_2 A = \rho g (y+L) A$$

Total upthrust = F_B = $\rho g (y+L) A - \rho g y A$

LA = volume of the cylinder.

<u>Archimedes Principle</u> states that the upthrust (upward vertical force due to the fluid) on a floating body immersed in a fluid is equal to the weight of the fluid displaced by the floating body.

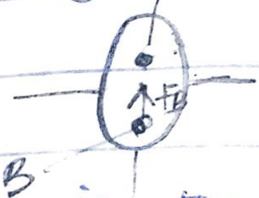
Conditions of Equilibrium

<u>Stable equilibrium</u> — A small displacement from the position of equilibrium produces a moment to restore the body back to the position of equilibrium.

<u>Unstable equilibrium</u> — A small displacement results in an overturning moment tending to

displace the floating body further from the position of equilibrium.

<u>Neutral Equilibrium</u> - The floating body remains at rest following the displacement on the body.



<u>Metacentric height</u> = This is the distance between the centre of gravity of a floating body and the Metacentre M. The metacentric height gives the measure of the floatation stability of the body.

$$GM = MB \pm GB$$

$$= \frac{I_o}{Vol} \pm GB.$$

$B = I_o / Vol.$

A floating body is stable if the centre of gravity is below the metacentre, otherwise, the body is unstable

A submerged body is stable if the centre of gravity is below the centre of Buoyancy.

Examples

Ship of total mass = 5,000,000 kg.

Mass of cargo = 25,000 kg

$I_o = 6,000 \, m^4$.

distance = 6m

GM = Metacentric height

$\theta = 5^\circ$

(i) Metacentric height (GM)

(ii) Distance between COG & COB (GB)

Solution

Overturning moment caused by shifting of cargo = righting moment of the ship

(Weight of cargo x distance moved) = (Weight of ship x GM x θ)

Weight of cargo = (mass of cargo x g)

$= 25000 \times 9.81 = 245250 \, N$

distance moved = 6m.

Weight of ship = (mass of ship x g)

$= 5,000\,000 \times 9.81$

$= 49050000 \, N$.

$\theta = 5^\circ = \frac{5}{360} \times 2\pi =$

(Weight of cargo x distance moved) = (weight of ship x GM

$(25000 \times 9.81 \times 6) = (5000000 \times 9.81 \times GM \times$

$\frac{5}{360}$

$$GM = \frac{25000 \times 9.81 \times 6}{5000000 \times 9.81 \times \left(\frac{5}{360} \times 2\pi\right)}$$

$GM = 0.344m.$

(b) $GM = MB \pm GB$

where GB = distance between COG and Centre of Buoyancy

$\therefore GB =$ ~~GM - MB~~ $MB - GM.$

$$MB = \frac{I_o}{Vol}$$

$I_o = 6000m^4$

volume of displaced seawater

from $\rho = \frac{m}{V}$

$$Vol = \frac{\text{mass of ship}}{\text{Density of water}} = \frac{5,000\,000}{1025}$$

$= 4878.05m^3$

$$MB = \frac{I_o}{vol.} = \frac{6000\ m^4}{4878.05m^3} = 1.23m.$$

$GB - MB - GM$

$= 1.23 - 0.344$

$= \underline{0.886m}$

(2) Total mass of ship = 700,000 kg

mass of cargo = 2,500 kg

distance moved by cargo = 6m

$\theta = 5^\circ = \frac{5}{360} \times 2\pi = \frac{1}{36}\pi$

$I_0 = 6000m^4.$

Solution

Overturning moment caused by shifting of cargo = Righting moment of the ship

(Weight of cargo x distance moved by cargo) = (Weight of ship x GM x θ)

$= \left(\overset{2500}{\cancel{700\,000}} \times 9.81 \times 6\right) = \left(700000 \times 9.81 \times GM \times \frac{\pi}{36}\right)$

$GM = \frac{2500 \times 9.81 \times 6}{700,000 \times 9.81 \times \left(\frac{\pi}{36}\right)} = \underline{0.246m}$

Depth of water displaced by carrier

$\rho = \frac{mass}{volume}$

$Volume = \frac{Mass}{density\ \rho}$

$\frac{800}{1025} = 0.780\ m^3$

$vol = Ah, \quad h = \frac{vol}{A}$.

~~Volume = Ah~~

$\text{Depth of water displaced} = \frac{\text{Volume}}{\text{Surface Area}}$

$h = \frac{0.780}{(1.5 \times 1.2)}$

$h = 0.43\ m$.

Carrier stable or unstable

Height of Centre of buoyancy above base

$= 0.5 \times d = 0.5 \times 0.43 = 0.215\ m$.

Height of COG above base $= 0.98\ m$.

$BG = 0.98 - 0.215 = 0.765\ m$

$BM = \frac{I_o}{vol} = \frac{0.16}{0.78} = 0.205\ m$

$BG = 0.765$

$BM = 0.205$

$GM = MB - GB = -0.55\ m$

$= 0.205 - 0.765$

The negative value of metacentric height indicates that the carrier is unstable.

(iii) Weight of carrier = $800 \times 9.81 = 7848\,N$

Weight of water displaced = $1025 \times 0.78 \times 9.81 = 7843$

Approximately equal, so carrier is floating.

(4) Mass of carrier = 100 Kg

Surface Area dimensions = $1.5 \times 1.2\,m \times 2.0\,m$

COG = 0.98 m

(i) Depth of water displaced

$$\rho = \frac{mass}{volume}$$

$$Vol. = \frac{mass}{\rho}$$

$$= \frac{100}{1025} = 0.098\,m^3$$

volume = Area x height (depth)

$$h = \frac{Volume}{Area} = \frac{0.098}{(1.5 \times 1.2)}$$

$h = 0.054\,m.$

$0.054 \times 0.5 = 0.027\,m$

$GB = 0.98 - 0.027 = 0.953\,m$

$MB = \frac{I_o}{V} = \frac{0.16}{0.098} = 1.632\,m$

$\therefore MB = GB + MG$

$MG = MB - GB$

$= 1.632 - 0.953$

$GM = 0.68\,m$

The positive value of GM indicates that the carrier is stable.

(iii) Weight of carrier $= 100 \times 9.81 = 981N$

Weight of water displaced $= \rho \times 0.098 \times 9.81$

$= 985.41$

~~Approximately stable~~, ~~so the carrier is floating~~.

Rayleigh's method

$R = f(L, V, \mu\ \rho \cdot K)$

Introduce a dimensionless constant A.

$R = A(L\ V\ \mu\ \rho\ K)$

$R = \text{Force} = \text{Kg m/s}^2 = [MLT^{-2}]$

$L = \text{Length}\ [L]$

$V = \text{Velocity m/s} = LT^{-1}$

$\mu = \text{Air viscosity} = ML^{-1}T^{-1}$

$\rho = \text{Density} = \frac{m}{V} = ML^{-3}$

$K = \text{Bulk modulus} = ML^{-1}T^{-2}$.

$R = A\,L^a V^b \mu^c \rho^d K^e$

$R = A[L]$

$[MLT^{-2}] = A[L]^a[LT^{-1}]^b[ML^{-1}T^{-1}]^c$

$[ML^{-3}]^d\,[ML^{-1}T^{-2}]^e$

comparing coefficients.

M: $c + d + e = 1$ —— (1)

L: $a + b - c - 3d - e = 1$ —— (2)

T: $-b - c - 2e = -2$ —— (3)

from (1), $d = 1 - c - e$ — (4)

from (3) $b = 2 - c - 2e$ — (5)

from (2), $a = 1 - b + c + 3d + e$ — (6)

Substitute (4) and (5) into (6)

$$a = 1 - (2 - c - 2e) + c + 3(1 - c - e) + e$$

$$1 - 2 + c + \cancel{2e} + c + \cancel{3} - 3c - \cancel{3e} + \cancel{e}$$

$$a = 2 - c$$

$$R = A\left(L^a V^b \mu^c \rho^d K^e\right)$$

$$\cancel{R = A(L^a)(LT^{-1})^b (ML^{-1}T^{-1})^c}$$

$$R = A(L)^{2-c}(V)^{2-c-2e}(\mu)^c(\rho)^{1-c-e}K^e$$

$$R = A\left(L^2V^2\rho\right)\left(L^{-c}V^{-c}\mu^c\rho^{-c}\right)\left(V^{-2e}\rho^{-e}K^e\right)$$

$$R = A(LV)^2\rho\left(\frac{\mu}{LV\rho}\right)^c\left(\frac{K}{V^2\rho}\right)^e$$

$$\therefore A\rho L^2V^2\ \phi\left[\frac{\mu}{\rho LV}\right]\left[\frac{K}{V^2\rho}\right]$$

Buckingham π

$D = f(W, h, \mu\ \rho\ V)$

No of variables = 6

No of fundamental dimensions = 3

No of π terms = (No of variables - fund. dimen)

= 6 - 3

= 3 π terms

Repeating variables = W, ρ, V

D = Force = ~~MLT~~ MLT^{-2}

W = width = L

h = height = L

μ = Viscosity = $ML^{-1}T^{-1}$

ρ = ML^{-3}

V = velocity = LT^{-1}

$\pi_1 = D\, W^a \rho^b V^c$

$\pi_2 = h\, W^a \rho^b V^c$

$\pi_3 = \mu\, W^a \rho^b V^c$

for π_1, $M^0L^0T^0 = D\,W^a \rho^b v^c$

$$M^0L^0T^0 = MLT^{-2} \cdot (L)^a (ML^{-3})^b (LT^{-1})^c$$

comparing variables/powers,

M : $1 + b = 0$ — (1)

L : $1 + a - 3b + c = 0$ — (2)

T : $-2 - c = 0$ — (3)

from (1), $b = -1$

from (3), $c = -2$

Substitute b and c into (2).

$a = -2$
$b = -1$
$c = -2$

$$1 + a - 3(-1) + (-2) = 0$$

$$1 + a + 3 - 2 = 0$$

$$a + 2 = 0, \quad a = -2$$

$$\pi_1 = D W^{-2} \rho^{-1} v^{-2}$$

$$\pi_1 = \frac{D}{\rho v^2 W^2}$$

for π_2, $M^0L^0T^0 = h\,W^a \rho^b v^c$

$$M^0L^0T^0 = L(L)^a (ML^{-3})^b (LT^{-1})^c$$

M : $b = 0$

L : $1 + a - 3b + c = 0$

T : $-c = 0$ $\quad a = 3b \cancel{-c} - 1$

$b = 0$,
$c = 0$,
$a = -1$

$\therefore \pi_2 = h\, W^a \rho^b V^c$

$= h\, W^{-1} \rho^0 V^0$

$\pi_2 = \frac{h}{W}$

$\pi_3 :\ \mu\, W^a \rho^b V^c$

$M^0 L^0 T^0 = M L^{-1} T^{-1} (L)^a (ML^{-3})^b (LT^{-1})^c$

$M: 1 + b = 0$

$L: -1 + a - 3b + c = 0$

$T: -1 - c = 0$

$b = -1$ $\qquad a = 1 + 3b - c$

$c = -1$ $\qquad a = 1 - 3 + 1$

$a = -1$

$a = -1$

$b = -1$

$c = -1.$

$\pi_3 = \mu\, W^{-1} \rho^{-1} V^{-1}$

$\pi_3 = \frac{\mu}{W \rho V}$

$\therefore \pi_1 = \phi(\pi_2, \pi_3).$

P.Q 16/17

① $D = f(L, \rho\ \mu\ u\ g)$

no of variables = 6

no of fundamental qtys = 3

no of π-terms = $(6-3) = 3$ π terms.

Repeating variables = ρ u, L

$\pi_1 = D\ \rho\ u\ L$

$\pi_2 = \mu\ \rho\ u\ L$

$\pi_3 = g\ \rho\ u\ L$

D = Force = MLT^{-2}

L = length = L

ρ = density = ML^{-3}

μ = Viscosity = $ML^{-1}T^{-1}$

u = Velocity = LT^{-1}

g = acc. gravity = LT^{-2}

$\pi_1 = D\ \rho^a\ u^b\ L^c$

$\pi_2 = \mu\ \rho^a\ u^b\ L^c$

$\pi_3 = g\ \rho^a\ u^b\ L^c$

$\pi_1 = MLT^{-2}\ (ML^{-3})^a\ (LT^{-1})^b\ L^c$

$M^0L^0T^0 = MLT^{-2}\ (ML^{-3})^a\ (LT^{-1})^b\ L^c$

M: $1 + a = 0$ $\qquad a = -1$

L: $1 - 3a + b + c = 0$ $\qquad 1+3-2+c = 0$

$2 + c = 0$

T: $-2 - b = 0$ $\qquad b = -2$ $\qquad c = -$

$\therefore a = -1$
$b = -2$
$c = -2$

$\pi_1 = D \rho^a u^b L^c$

$\pi_1 = D \rho^{-1} u^{-2} L^{-2}$

$= \dfrac{D}{\rho u^2 L^2}$

$\pi_2 = \mu \rho^a u^b L^c$

$M^0 L^0 T^0 = M L^{-1} T^{-1} (M L^{-3})^a (L T^{-1})^b L^c$

$M: 1 + a = 0$

$a = -1$

$L: -1 - 3a + b + c = 0$

$T: -1 - b = 0 \quad b = -1$

$a = -1$
$b = -1$
$c = -1$

$-1 + 3 - 1 + c = 0$

$1 + c = 0$

$\pi_2 = \mu \rho^{-1} u^{-1} L^{-1} \quad c = -1$

$\pi_2 = \dfrac{\mu}{\rho u L}$

$\pi_3 = g \rho^a u^b L^c$

$M^0 L^0 T^0 = L T^{-2} (M L^{-3})^a (L T^{-1})^b L^c$

$M: a = 0$

$L: 1 - 3a + b + c = 0$

$T: -2 - b = 0 \quad b = -2$

$a = 0$
$b = \cancel{0}\,-2$
$1 - 0 + \cancel{0}\,{-2} + c = 0$
$1 + c = 0$
$-1 + c = 0 \quad c = \cancel{0}$
$c = 1$

$$\pi_3 = g \rho^0 u^{-2} L^{1}$$

$$\pi_3 = \frac{gL}{u^2}$$

$$\pi_1 = \frac{D}{\rho u^2 L^2}$$

$$\pi_2 = \frac{\mu}{\rho u L}$$

from π_2, $\frac{1}{\pi_2}$

$\Downarrow$

Reynold's no $= \frac{\rho u L}{\mu}$

$$\pi_3 = \frac{gL}{u^2}$$

~~Froude's~~ number =

from $\pi_3 = \frac{gL}{u^2}$

$$\pi_3 u^2 = gL$$

$$\pi_3 u = \sqrt{gL}$$

$$\pi_3 = \frac{\sqrt{gL}}{u}$$

Froude's no $= \frac{1}{\pi_3} = \frac{u}{\sqrt{gL}}$

for dynamic similarity, Re should be equal.

$$\left[\frac{\rho V L}{\mu}\right]_p = \left[\frac{\rho V L}{\mu}\right]_m$$

$$\left[\frac{VL}{\cancel{\mu}}\right]_p = \left[\frac{VL}{\cancel{\mu}}\right]_m$$

$$V_p = \frac{\cancel{\mu_p} L_m}{\cancel{\mu_m} L_p} V_m$$

$$V_p = \frac{0.018 \times 10^{-4} \times 3}{0.012 \times 10^{-4} \times 300} \cdot 30 = 0.2\,m/s$$

Since resistance = mass × Acc

$$= \rho L^2 V^2 \qquad \rho = \frac{m}{v}$$

$$m \sim \frac{\rho}{v}$$

$$\frac{F_p}{F_m} = \frac{(\rho L^2 V^2)_p}{(\rho L^2 V^2)_m}$$

$$= \frac{1030}{1.24}\left(\frac{300}{3}\right)^2\left(\frac{0.2}{30}\right)^2$$

$$= 369.17$$

$$F_p = 369.17 \times 60 = 22150.2\,N$$

A pipe of diameter 1.5m is required to transport oil of sp. gravity 0.90 and viscosity 3×10^{-2} poise at a rate of $300 L s^{-1}$. Tests were conducted on a 15 cm diameter pipe, using water at 20°C find the velocity and flow rate in the model.

<u>Prototype data</u>

$d_p = 1.5m$

$(sp.\ gravity)_p = 0.90$

$(viscosity)_p = \mu_p = 3 \times 10^{-2}$ poise

$Q_p = 300 L/s = 3 m^3/s.$

$\rho_p = 0.9 \times 1000 = 900 kg/m^3$

Model data

$d_m = 15 cm = 0.15 m$

$\rho_m = 1000 kg/m^3$

$V_m = ?$

$Q_m = ?$

$\mu_m = 1 \times 10^{-2}$ poise

From Reynold's members

$$\left[\frac{\rho V L}{\mu}\right]_{prototype} = \left[\frac{\rho V L}{\mu}\right]_{model}$$

where $L = D$

$$\therefore \frac{\rho_p V_p D_p}{\mu_p} = \frac{\rho_m V_m D_m}{\mu_m}$$

$Q = AV$

~~$Q_m = A_m V_m$~~

$Q_p = A_p V_p$

$V_p = \frac{Q_p}{A_p} = \frac{3}{1.77}$

$A_p = \frac{\pi d^2}{4} = \frac{\pi \times 1.5^2}{4}$

$= 1.77$

$Q_p = A_p V_p$

$V_p = \frac{Q_p}{A_p}$

$V_p = \frac{3 m^3/s}{1.77 m^2} = 1.697 \ m/s.$

$\frac{\rho_p V_p . D_p}{\mu_p} = \frac{\rho_m V_m D_m}{\mu_m}$

$\frac{900 \times 1.697 \times 1.5}{3 \times 10^{-2}} = \frac{1000 \times V_m \times 0.15}{1 \times 10^{-2}}$

$V_m = \frac{900 \times 1.697 \times 1.5 \times 1 \times 10^{-2}}{1000 \times 0.15 \times 3 \times 10^{-2}}$

$V_m = 5.091 \ m/s$

$Q_m = A_m V_m$

$Q_m = \frac{\pi d^2}{4} \times 5.091$

$= \frac{\pi \times 0.15^2}{4} \times 5.091$

$= 0.0899 \ m^3/s.$

Ship 300m long

Prototype data

$L_p = 300m$

$\rho_p = 1030 kg/m^3$

$\mu_p = 0.018$ stokes $\rightarrow 0.018 \times 10^{-4} m^2/s$

$V_p = ?$

$F_p = ?$

Model data

$\frac{L_p}{L_m} = 100.$

$\frac{300}{L_m} = \frac{}{100}$

$L_m = 3m$

$V_m = 30m/s$

F_m 60N

$\mu_m = 0.012$ stokes

$\rho_m = 1.24 kg/m^3$

For dynamic similarity, The ~~froude~~ Reynold's no must be the same in prototype and model.

$$\left(\frac{\rho V L}{\mu}\right)_p = \left(\frac{\rho V L}{\mu}\right)_m$$

$$\left(\frac{V L}{\mu}\right)_p = \left(\frac{V L}{\mu}\right)_m.$$

$$\frac{V_p L_p}{\mu_p} = \frac{V_m L_m}{\mu_m}$$

$$V_p = \frac{\mu_p V_m L_m \times 1.24}{L_p \mu_m \times 1030} = \frac{0.018 \times 10^{-4} \times 30 \times 3}{0.012 \times 10^{-4} \times 300}$$

0.45

Resistance = mass × Acc

$= \rho L^2 V^2$

$(\rho L^2 V^2)_p = (\rho L^2 V^2)_m$

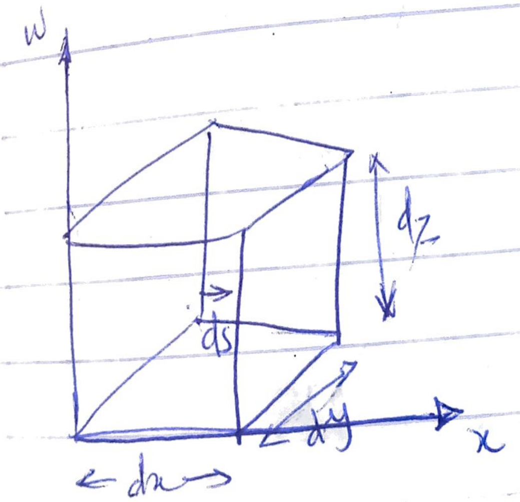
$$\frac{F_p}{F_m} = \frac{\rho_p (L_p)^2 (V_p)^2}{\rho_m (L_m)^2 (V_m)^2}$$

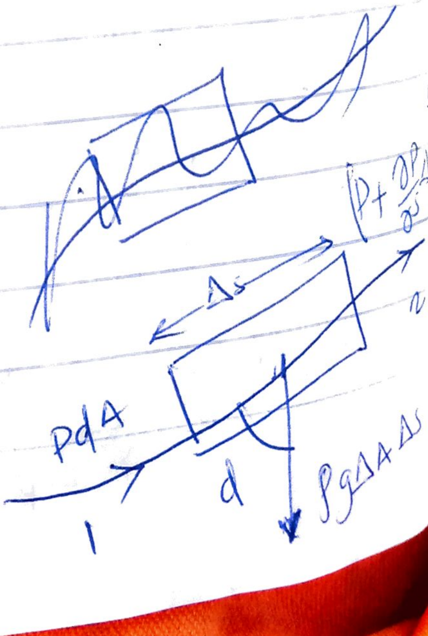
$$\frac{F_p}{60} = \frac{1030 \times 300^2 \times (0.2)^2}{1.24 \times (3)^2 \times (30)^2}$$

$$\frac{F_p}{60} = 369.176$$

$F_p = 60 \times 369.176$

$F_p = 22150.54\ N.$





dA = cross-sectional area of fluid element

ds = Length of fluid element

dW = Weight of fluid element

P = Pressure on fluid element at point 1

$P + dP$ = pressure on the element at point 2.

v = velocity of fluid element.

External forces accelerating the fluid element in direction of the streamline

$$= PdA - (P + dP)\,dA$$

$$= -dP\,dA$$

Weight of fluid element

$$dW = \rho g \cdot dA \cdot ds$$

The component of the weight of the fluid element in the direction of flow

$$= \rho g \cdot dA \cdot ds \cos\theta$$

$$= \rho g\, dA \cdot ds \cdot \frac{dz}{ds}$$

Flow rate $Q = A V$ $\qquad V_2 = ?$

$V_1 = 3 \, m/s$

$Q = A \cdot V$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (d_1)^2}{4}$$

$$= \frac{\pi \times \cancel{2}^2}{\cancel{4}}$$

$$Q = 3 \times \pi = 9.424 \, m^3/s$$

$$Q = A V_2$$

$$9.424 = \frac{\pi (d_2)^2}{4} \cdot V_2$$

$$9.424 = \frac{\pi \times 9}{4} \cdot V_2$$

$$V_2 = \frac{4 \times 3\pi}{9\pi} = 1.33 \, m^3/s.$$

$$Re \ \frac{V D}{\mu}$$

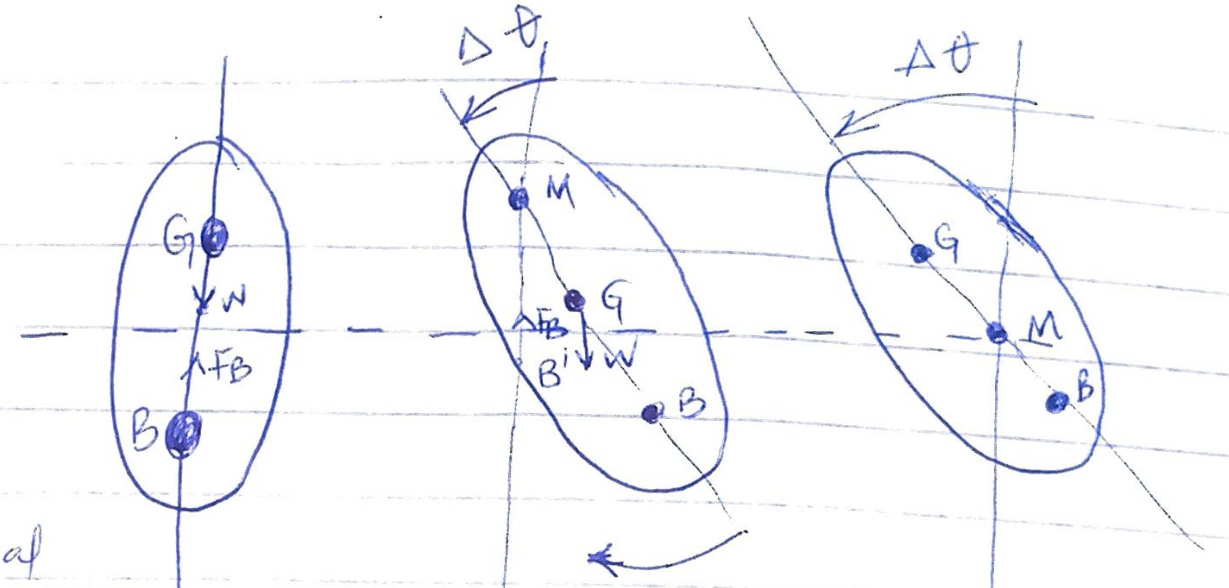
Laminar $2320 < Re$

Turbulent $2320 \geq Re$

$$Re_1 = \frac{V_1 D_1}{\mu} = \frac{3 \times 2}{1.24 \times 10^{-6}} = 4838.1 \times 10^{3}$$

$$Re_2 = \frac{V_2 D_2}{\mu} = \frac{3 \times 1.33}{1.24 \times 10^{-6}} = 3225 \times 10^{3}$$

Both Laminar



Neutral Equilibrium

untilted

↓

Remains at rest following the displacement on the body.

Stable Equilibrium

↓

Restoring moment

Overturning moment.

CVE 312

Surface & Ground Water Hydrology

Prof. Sunday Oyegoke.

chapter 1

- Hydrology defined
- History of hydrology
- Hydrological cycle. & hydrologic bodies

P = Precipitation T_s = Transpiration

R_1 =

R_2 =

R_g = ~~Infiltration~~

E_s = Evaporation

I = Infiltration.

$10,500\ mi^2$

- The continuous process by which water is transported from the oceans to the atmosphere to the Land and back to the sea.

The Hydrologic Budget - For any system, a water budget can be developed to account for the hydrologic components

The primary input in a hydrological budget
precipitation (rainfall)

T - Transpiration
E - Evaporation
P - Precipitation
R - Surface Runoff
G - Ground water flow
I = Infiltration.

26/05/2024

Chapter 2.

Precipitation

Amount of precipitable water = W

$$W = 0.004 \int q \, dP$$

Mean annual rainfall of states in Nigeria.

Time Get a plot of mean rainfall of Nigeria
" " " " evaporation/transpiration

self-recording rainfall

Areal Precipitation

Isohyetal rainfall

204

206

210

02/06/2021 Chapter 2 — Assignment

Probable Maximum Precipitation.

Ch. 3 - Interception and Depression Storage.

Ch. 4 - Infiltration

Ch. 5 - Evaporation & Transpiration

Ch. 6 - Streamflow

* Ch. 7 - Hydrologic Data Sources

* 8 - Instrumentation.

Rainfall data — meteorological departments,

Water Coorporations.

Research Stations eg IITA

Stream flow - Min. of Water resources.

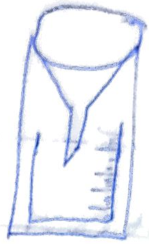
- Nigerian ~~Institute~~ Hydrological Services Agen (NIHSA)

- ~~Water~~ River Basin Development Authorities

- State Water Corporations.

- National Water Resources Institute

Ch. 8 - Instrumentation.

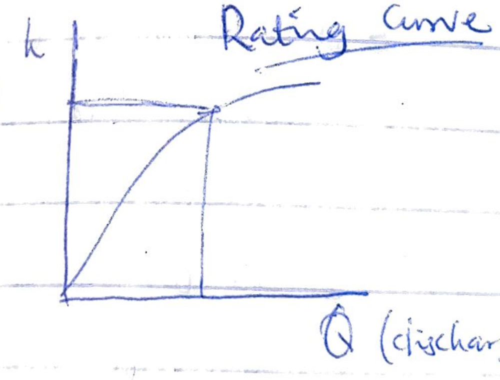


1. Manual Raingauge
2. Self reporting raingauge
3. Evaporimeter / Evaporating pan
   ↓
   Lysimeter
4. Thermometers (for temperature).
5. Weather station - Pressure, humidity, rainfall, temperature

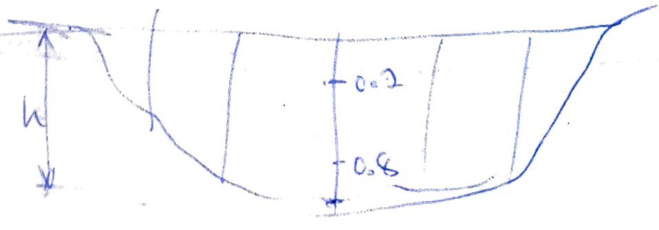
How is streamflow measured?

The depth of water / stream varies, depending on the position.

Rating Curve



Divide the river into various segments, currentmeter - to measure ~~Q (discharge)~~ velocity of the flow.



$$V_{average} \; (\bar{V}) = \frac{V_{0.2} + V_{0.8}}{2}$$

$$Q = \frac{vol}{time}$$

Add the velocity for each segment and find the average

16/06/2021

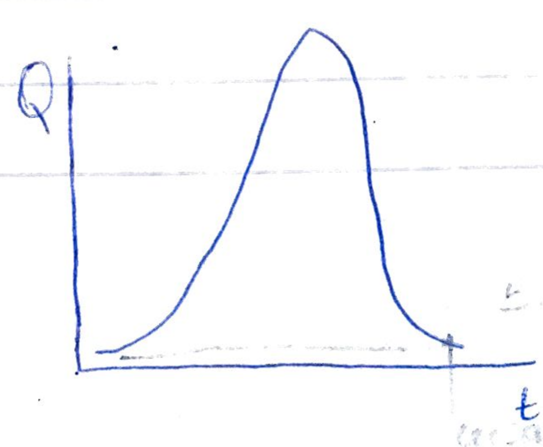
Chapter 11

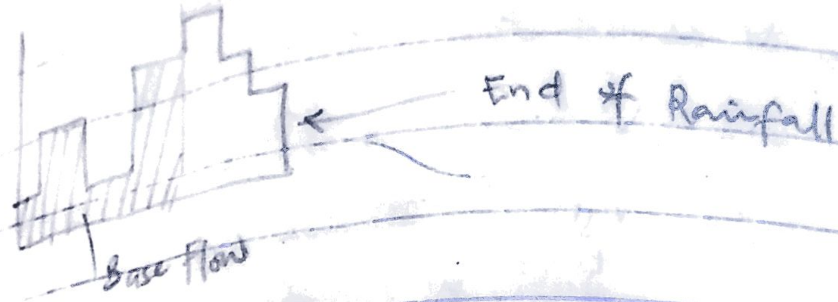
Hydrographs

- A graph of the discharge (rain) / waterfalls.
- The purpose of water-graph analysis.
  - To have adequate planning to know whether to store water during ~~droughts~~ rainy seasons so as to use during droughts

Current meter - to measure the flow of water on the stream.

11.4





The Base flow – ~~BF~~ flow coming from the ground water

* Base flow model.
* Hydrograph Separation Techniques.
* Summary of Time of Concentration formulas

A hydrograph is a continuous graph showing the rate of streamflow with respect to time, normally obtained by means of a strip recorder, which indicates stage vs time (stage hydrograph)

Assignment

- Plot the hydrograph
- Label the rising limb (conc curve) the crest segment and recession limb

(b) Determine the hour of cessation of the direct runoff using a semilog plot of Q vs time

(c) Use the base flow portion of your semilog to determine the groundwater recession constant.

(d) Carefully construct and label base flow separation curves on the graph of part a, using different methods

Time (hrs)	Q (cfs)	Time (hrs)	Q (cfs)
0	102	11	75
1	100	12	60
2	98	13	54
3	220	14	48.5
4	512	15	43.5
5	630		
6	460		
7	330		
8	210		
9	150		
10	105		

$q_t = q_0 K^t$ or

$q_t = q_0 e^{-kr}$

q_0 = specified initial discharge

q_t = discharge at any time t after flow q_0

K = recession constant

e = constant

$K^t = \frac{q_t}{q_0}$

$k = \left(\frac{q_t}{q_0}\right)^{\frac{1}{t}}$

23/06/2021.

Chapter 12

Unit Hydrographs

To know the flow that will cause flooding of rivers

To Estimate the maximum possible flood flow.

* Fig 12.1 (a) 2 hour rainfall effect lasted 30 hr

  (b) 12 hour ⇒ 40 hrs.

The shaded portion → Underground flow

Example 12-1

$$\frac{\Sigma(\Delta R \times \Delta t)}{\text{area}} = \frac{2447 \text{ cfs-hr}}{1716 \text{ ac}} = 1.4''$$

* Ex 12-2

* Ex 12-3

12.2, 12.18 } Assignment

S- Hydrograph Method.

Ex 12-4

12.5 Instantaneous Unit hydrograph

12.6 -

30 - 06 - 2021

Ch. 13

Hydrograph Routing.

Routing methods

Hydrologic Routing ✓

Hydraulic Routing

Inflow rate - Outflow rate = Rate of change of storage.

$$I - O = \frac{ds}{dt}$$

13.6 Assignment

(1) Muskingum method

Eqn 13.5

Example 13.1

$$O_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$$

$C_0 = 0.0477$

$C_1 = 0.428$

$C_2 = 0.524$.

Ch. 26 - Probability & Statistics.

26.7
26.9 Assignment

Sd 6/7

$P(F) = 0.1$

$T = \frac{1}{P(F)} = \frac{1}{0.1} = 10$

$T = \frac{1}{P(F)} = \frac{1}{1 - P(\bar{F})}$

07-07-2021

Ch. 27 – Frequency Analysis

- How frequent a thing occurs

$x = \bar{x} + K s$ — SD

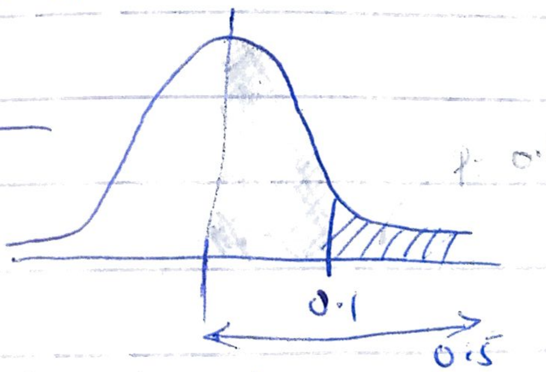
Mean

frequency factor (z)

$Q = \bar{Q} + z S_Q$

Ex. 27.2

Normal dist.



$P > 0.1$

Exceedance probability > 10 yrs

$F(z) = 0.5 - 0.1 = 0.4$

$z = 1.282$

$Q = \bar{Q} + z s_Q$

$Q_{10} = 14.9 + 1.282(5.9)$

$= 15.1$

Ex 27.3

Log normal table

Ex. 27.4)

(Gumbel)

$$K = -\frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\ln\left(\frac{T}{T-1}\right)\right)$$

$\bar{x} = 1000$

$s = 400$

$T = 25$ (once in 25 yrs)

$x = \bar{x} + Ks$

$$K = -\frac{\sqrt{6}}{\pi}\left(0.5772 + \ln\ln\left(\frac{25}{24}\right)\right)$$

$= 3.088$

$\therefore x = \bar{x} + Ks$

$= 1000 + (3.088 \times 400)$

$= 2235$ cfs

27.2

27.4

27.7

Ground Water & Wells

Perform a geophysical Survey to analyze ground water.

Equation 4.13.

$$Q = -\pi K \frac{H_0^2 - H_w^2}{\ln\left(\frac{r_0}{r_w}\right)}$$

* Example 4.3

K = Permeability

* Example 4.4

$$u = \frac{r^2 S}{4Tt}$$

$$W(u) = 8.62$$

$$s = -\frac{Q}{4\pi T} W(u)$$

4.6 - Groundwater Exploration

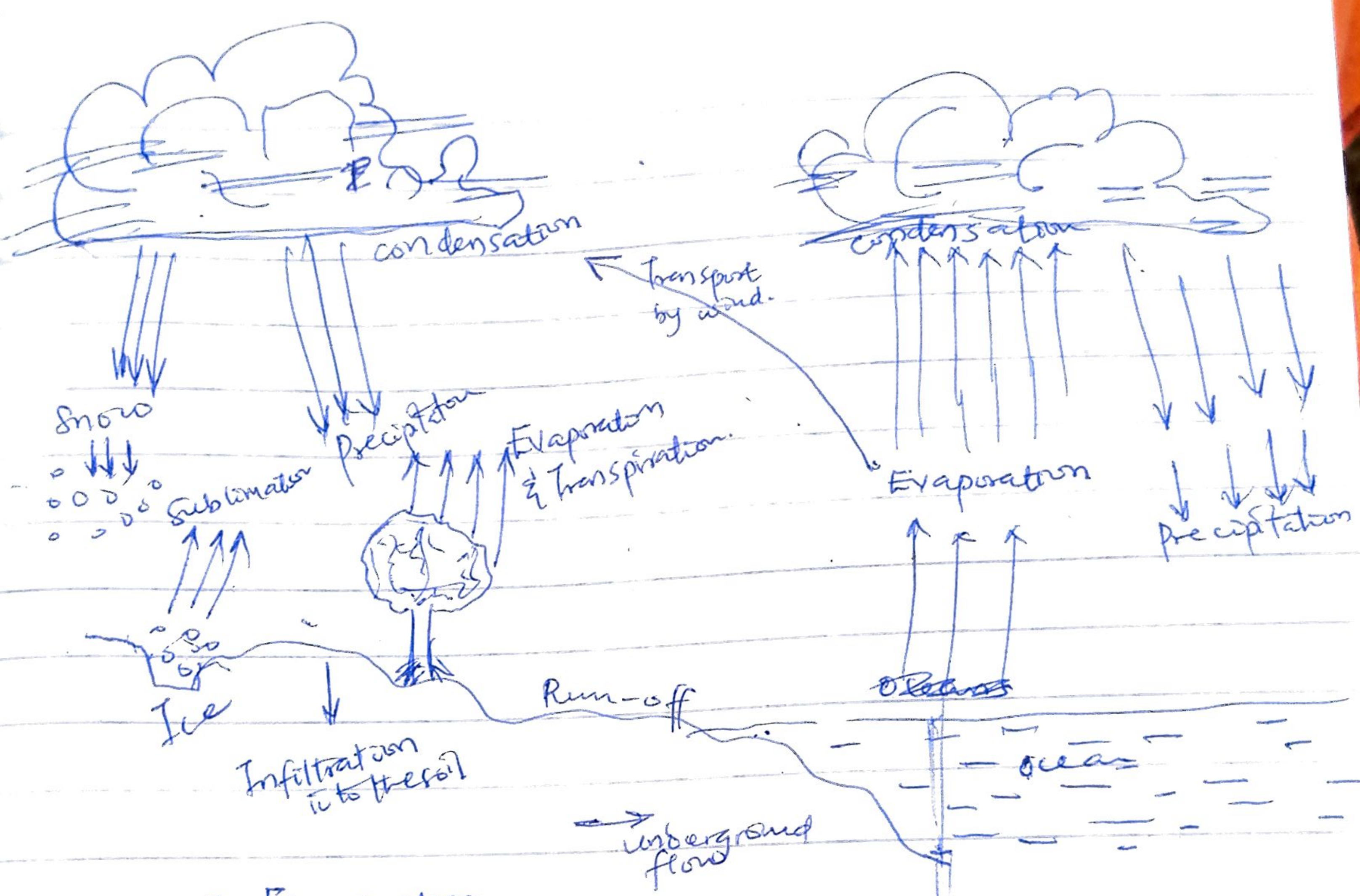
4.63 - Well logging methods

4.64 - Test drilling.

4.9 - Methods of well digging

'Rotary drilling

- Precipitation — Source equipments for precipitation
- Hydrographs
- Normal Distributions
- Statistics — Regression flow
- Groundwater — Boreholes



1. Evaporation

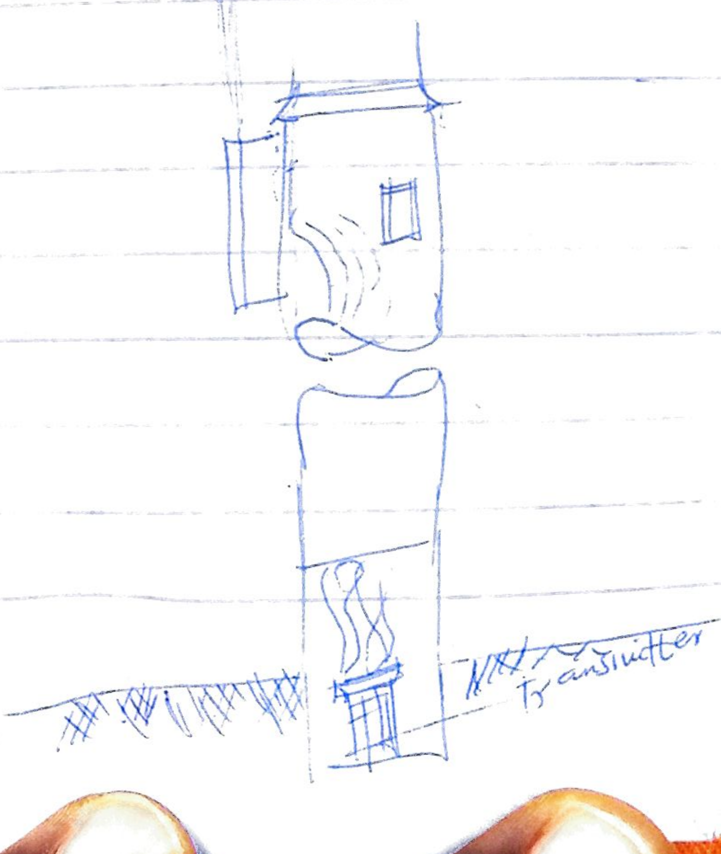
(2) Precipitation

(3) Condensation

(4) Run-off

(5) Evapotranspiration

(6). Infiltration



Hydrology is an earth Science that deals with the occurrence, distribution, movement and properties of the waters of the earth.

- Design & operation of water resources
- Waste water treatment.
- Irrigation
- Flood Risk Management
- Navigation
- Hydropower
- Ecosystem Modelling
- Pollution Control.

Ch. (1) Hydrology & the Hydrologic cycle

Ch. 2 Precipitation
- forms, - rain, snow
- formation - coalescence
  - Ice - crystal process.
- types: Convective
  Cyclonic
  Orographic.
~~Areal~~ precipitation
- Arithmetic av.
- Thiessen method
- Isohyetal.

Ch. 3, 4, 5 . ~

<u>Ch. 7</u> - Hydrologic Data Sources