ATTENDANCE IS COMPULSORY (10 MRKS)
ASSIGNMENT (10 MRKS)
TEST (20 MRKS)

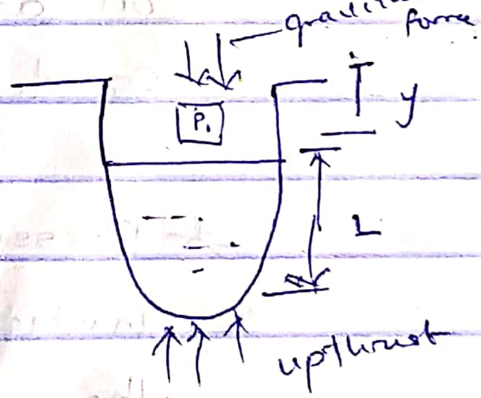
FLUID MECHANICS II

COURSE CONTENT

* Floatation and stability
* Dynamics of fluid flow
* Conservation equation of Mass & Momentum
* Euler and Bernolis Equation
* Introduction to Incompressible viscous flow
* Reynold's Number
* Dimensional Analysis
* Hydraulic Models simplicity
* Flow's meter and Error in Measurement

Hydrostatic is the study of fluid at rest.

Equipment:
Piezometer
The Barometer
Micro Manometer



Fluid

~~Gravitat~~ force of Gravity is natural

Modelling

Hydrostatics studies fluid at rest

From our everyday experience, we have become familiar with the fact that matter occur in 3 different forms: the three state are (1) solid

(2) liquid

(3) gases

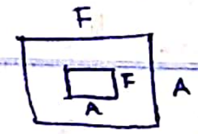
Under ordinary condition, stones, iron, copper and chalks are solids. Liquids under normal condition has example like water, oil and mercury while Oxygen, carbon (iv) Oxide, hydrogen are gases.

* Pressure :- there's a difference in a manner in which a force is applied to a fluid and the way it is applied to a solid. A force can be supported by a single point of a free solid but it can only be supported by a surface of enclose fluid, If a force F is applied to a surface of fluid, and it acts over the area of a perpendicular to it, then the average pressure:

$$P = \frac{\text{Force}}{\text{Area}} \quad N/m^2 \quad \text{Pounds}/\text{Sq.Ft}$$

When a fluid is under pressure, it exert a force on a surface, which contains the fluid while relating the magnitude of force exerted to the pressure and area, it must be accompany by a statement indicating direction of the force.

$$P = \frac{\Delta F}{\Delta A}$$

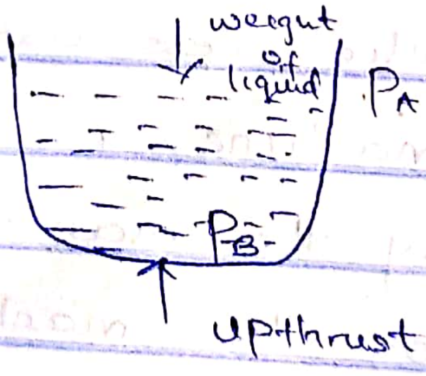


$$\Delta F = P \times \Delta A$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Density as indicated as (ρ) is mass per unit volume, the mass of homogenous body of material of volume $V = \rho v$. The unit of density is expressed appropriate to the system being used. kg / cm^3

PRESSURE DUE TO A WEIGHT OF A LIQUID



$$P_b A - P_a A = A h \rho g$$

PRESSURE IN A CONFINED

In addition to a pressure due to its weight, a confined liquid subjected to external pressure by the application of external forces. Suppose the liquid is in a cylinder and a tight fitting piston is placed on the surface of the liquid, if a force F is applied to the piston, it will ~~remain~~ remain in practically the same position, since the possibility of liquid is very small. If A is the area of the piston, the external force produces a pressure $P = F/A$ at the surface of the liquid

(No atmospheric pressure)

For an hydraulic press, that is design to lift a weight $W = PA$ which can equally be expressed as PA i.e $F\frac{A}{a}$ the hydraulic press can be considered as a simple machine, in which the force exerted by the machine divided the force exerted on the machine is equal to the ratio of the area i.e $\frac{W}{F} = \frac{A}{a}$

MEASUREMENT OF PRESSURE

Assignment (LATEST TUESDAY)

Show that the pressure head in a body of fluid (water) with pressure p measured using a mercury-manometer is given by $\frac{p}{\rho_w g} = \left[\left(\frac{\rho_m}{\rho_w}\right) h_2\right] - h_1$

where p = pressure in the body of fluid,

ρ_w = density of water

g = acceleration due to gravity

ρ_m = density of mercury

h_2 = height of column of mercury on right limb above the datum

h_1 = height of column of mercury on left limb of the manometer

Fluid Mechanics by Timo Shenko

TOD

Buoyancy Forces | upthrust.

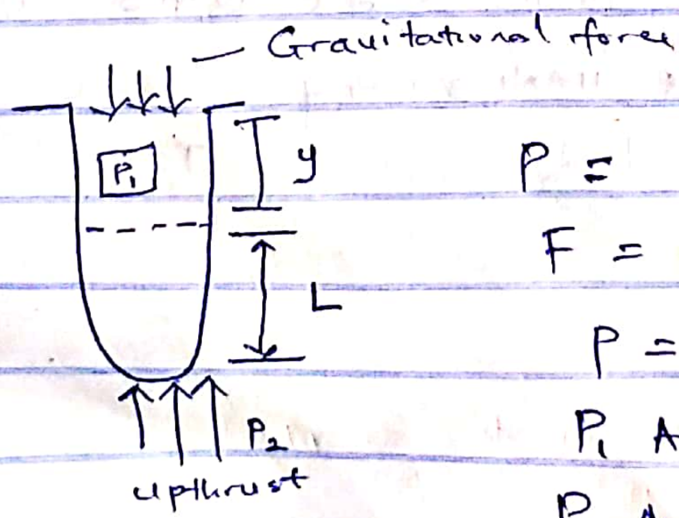
This is an upword force exerted by a fluid that opposes the weight of a partially or fully immersed object. This is as a result of differences in pressure acting on opposite side of an object immersed in a state of fluid or in a static fluid.

Factor affecting Buoyant Forces | upthrust

i. Density of the fluid
ii. Volume of the fluid
iii. Local acceleration due to gravity

Examples

← Application of Buoyant Force In Gasline in waterlog Area



$P = F/A$

$F = PA$

$P = \rho g h$

$P_1 A = \rho g y A$

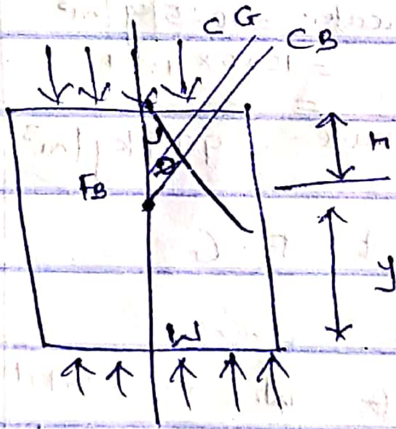
$P_2 A = \rho g (y + L) A$

Conditions for Equilibrum

① for a stable Equilibrum :- a small displacement from the position of Equilibrum produces a moment to restore back the body to position of Equilibrum.

② Unstable Equilibrum :- is a condition in which small displacement result in an over turning moment tending to displace the floating body further to the position of Equilibrum.

③ Neutral Equilibrum :- the floating body remain at rest following a displacement at equilibrum of a body.



Example :- A ship of total mass 5000000kg floating on the sea in a stable condition a portion of the cargo of mass 25000 kg ship through a distance of 6m at right angle to the vertical plane of the longitudinal ~~angle~~ of the ship ~~cargo~~ causing the ship to heel through an angle of 5°. If the

(i) The metacentric height

second moment of inertia is $6000\,m^4$, calculate

(ii) The distance between the center of Gravity and Buoyancy, giving the coeff to be $1025\,kg/m^3$, density of raw water to be $999\,kg/m^3$.

Solution

Total mass of ship = 5000,000kg

Weight of the ship = 5000,000 × 9.81 = 49050KN

Mass of Cargo = 25000 × 9.81 = 245250

Distance shift through the cargo = 6m

Angle heel through by the cargo = 5°

Second moment of inertia = $6000\,m^4$

Density of sea water = $1025\,kg/m^3$ = 1025 × 9.81 =

Density of raw water = $999\,kg/m^3$

(i) Find (i) Gm & B.G

$Gm = \frac{W_1 x}{W \tan\theta}$ where,

$W_1 = 245.25\,KN$

$W = 49050\,KN$

$x = 6m$

$\theta = 5$

$Gm = ?$

$= \frac{245.25 \times 6}{49050 \tan(5)}$

$Gm = \frac{245.25 \times 6}{49050 \times \tan 5} = \frac{1471.5}{49050 \times 0.0875}$

$Gm = \frac{1471.5}{4291.3189}$

$= 0.3429m$

Recall that

$Gm = \frac{I}{V} - BG$

Volume of fluid displace = $\frac{\text{Weight of ship}}{\text{Weight density of } H_2O}$

$V = \frac{49050}{10.05525}$

$= 4878.04878\,m^3$

$Gm = \frac{6000}{4878.04878} - BG$

$\rightarrow 0.3429 = 1.23 - BG$

$0.3429 - 1.23 = -BG$

$BG = 0.8871m$

15TH OCTOBER 2021

EULER EQUATION

Euler equation or steady flow of an ideal fluid along the stream line is a relation between the ~~vel~~ velocity, pressure, and density of a moving fluid. It is based on Newton's second law of motion $F = ma$. The integration of Euler Equation resolve into Bernoulli's equation in the form of energy per unit weight of the fluid

Assumptions

1. That the fluid is non-viscous i.e the frictional loss in that fluid.
2. That the fluid is homogenous and in-compressible i.e Mass, Density of the fluid is constant.
3. That the flow is continous and steady along the stream line.
4. That the velocity of the flow is uniform over the sections.
5. That no energy or force is involved in the flow except Gravitational force and pressure force.

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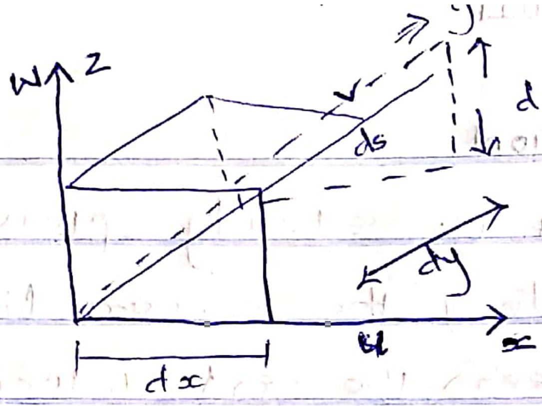
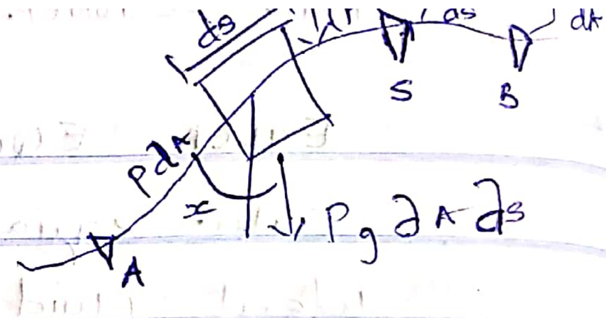
$CAH = \frac{Adj}{Hyp}$





dA = cross - sectional area of fluid element
dx = length of fluid element
dW = Weight of " "
P = Pressure on the element at point A
p + dp = pressure on the element at point B
v = Velocity of the fluid element.

The external forces tending to accelerate the fluid element in the direction of stream-line is equal to

$$= P\,\partial A - (p + \partial p)\, dA$$

$$= -\partial p \cdot \partial A$$

$$dW = \rho g\, dA \cdot ds$$

$$= \rho g\, dA\, ds \cos\theta$$

$$= \rho g\, dA\, ds\, \frac{dz}{ds}$$

$$= \rho g\, dA\, dz$$

Mass of fluid element $= \rho\, dA\, ds$

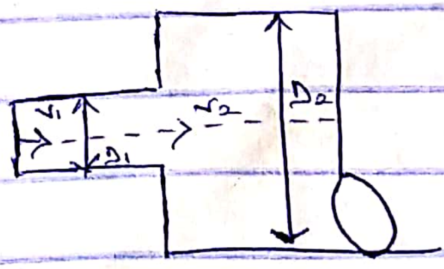
$$\frac{dv}{dt} = a = \frac{dv}{ds} \times \frac{ds}{dt} \quad v = V\frac{dv}{ds}$$

$$F = ma$$

$$-\partial p \cdot dA = \rho\, dA\, ds \times v\frac{dv}{ds}$$

$$\left[\frac{dp}{\rho} + g \cdot dz = -v\,dv\right] \text{— Euler}$$

Example :— On a circular conduit, there are different diameter $D_1 = 2m$, $D_2 = 3m$. The velocity in the entrance profile was measured as $V_1 = 3m/s$. Calculate the discharge and new velocity at the Outlet profile, determine also the type of flow in both conduit profile i.e whether it is a laminar or turbulent, taking temperature of water to be 12°C and density of water to be 999 kg/m³.



force = m×a
= kg m/s^2
= (MLT^{-2})

27TH OF OCTOBER 2021

Force
Length
Time

Dimensional Analysis

Fundamental quantities.

Length – M
Time – (s) T
Mass – kg M

velocity = m/s = LT^{-1}
Acceleration = m/s^2 = LT^{-2}
Density = kg/m^3
= ML^{-3}

Dimensional Analysis is a mathematical techniques taking use of studying dimension. In dimension Analysis, one force predict the physical parameters that will influence the flow and then by grouping this parameter in dimensionless combination in better understanding of the flow phenomenon is made possible.

Flow Characteristics, Units & Dimensions

Characteristics		Unit (SI)	Dimension MLT	Dimension FLT
Geometry –	Length	m	L	—
	Area	m^2	L^2	—
	Volume	m^3	L^3	—
Kinematics –	Time	s	T	
	Velocity	m/s	LT^{-1}	
	Acceleration	m/s^2	LT^{-2}	
	Discharge or flow rate	m^3/s	L^3T^{-1}	

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X-teristics	Unit (SI)	Dimension MLT	Dimension FLT
Dynamic – Mass	Kg	M	$FL^{-1}T^{2}$
– Force	N (kgm/s²)	MLT^{-2}	F
– Pressure	Pa (kgm⁻¹s⁻²)	$ML^{-1}T^{-2}$	FL^{-2}
– Energy	J (N-m)	$ML^{2}T^{2}$	$FL^{2}T^{2}$
– power	W (N-m/s)	$ML^{2}T^{-3}$	$FL^{2}T^{-3}$

Method of dimensional Analysis

If the number of variables involved in a physical phenomenon are known, then the relation among the variable can be determine by two methods

1. Rayleigh's method
11. Buckingham's π Theorem.

Rayleigh's Method

This is used for determining expression for a variable, that variable is dependent which depend on maximum of three to four variable, if the number of independent variable are more than 4, it is extremely difficult to obtain expressions for dependent variables.

Examples :- The resisting force R of a supersonic plane aircraft during flight can be considered as dependent upon the length of the aircraft, velocity v, air density, M and bulk modulus K. Express the function relationship btw the variable and resisting force.

$$R = f[L, V, M, \rho, K]$$

Solution

$R = f(l, v, ? ? K)$? $(R, l, v, ?, ?, K) = 0$

Number of variables $n = 6$ Number of primary variables, $m = 3$

Number of ? terms $= n - m = 6 - 3 = 3$ $f(? 1, ? 2, ? 3) = 0$

Step 2 :- Assume l, v and ? to be the repeating variables

$? 1 = l^x, v^y, ?^z, R$

$M^0 L^0 T^0 = [L]^x [LT^{-1}]^y [ML^{-3}]^z [MLT^{-2}]$

$M^0 L^0 T^0 = [$

R = Resultant force $\rightarrow MLT^{-1}$

L = length $\rightarrow L$

v = velocity $- ML^{-1}T^{-1}$

μ = air viscosity $- ML^{-1}T^{-1}$

ρ = density $- ML^{-3}$

K = Bulk modulus $- ML^{-1}T^{-2}$

$$MLT^{-1} = A(L), (LT^{-1}), (ML^{-1}T^{-1}), (ML^{-3}) (ML^{-1}T^{-2}$$

a, b, c, d, e represent powers

$$ML^{1}T^{-2} = A(L)^a \cdot (LT^{-1})^b, (ML^{-1}T^{-1})^c, (ML^{-3})^d (ML^{-1}T^{-2})^e$$

$M \Rightarrow 1 = c + d + e$ --- (i)

$L \Rightarrow 1 = a + b - c - 3d - e$ --- (ii)

$T \Rightarrow -2 = -b - c - 2e$ --- (iii)

making subject of formula

Solve simultaneously

$d = 1 - c - e$

$b = 2 - c - 2e$

$a = 1 - b + c + 3d + e$

$a = 1 - [2 - c - 2e] + c + 3[1 - c - e] + e$

$1 - 2 + c + 2e + c + 3 - 3c - 3e + e$

$a = 2 + 2c - 3c$

$a = 2 - c$

$R = f(L, v, \mu, \rho, K)$

$= A(L)^a (v^b)(\mu^c)(\rho^d)(K^e)$

$R = A[L^{2-c}][v^{2-c-2e}]\mu^c[\rho^{1-c-e}][K^e]$

collect like terms

$= A l^2 v^2 \rho^1 [L^{-c} v^{-c} \mu^c \rho^{-c}] [v^{-2e} \rho^{-e} K^e$

π

Buckingham's π Theorem

→ The velocity of propagation of a pressure wave is expected to depend on the elasticity of the liquid represented by the bulk modulus K and mass density ρ. Establish by dimensional analysis the possible relationship between the variables.

→ A thin rectangular plate having a width ω and height h is located at its normal to a moving stream of fluid. Assume the drag force D is a function of ω and h, the fluid viscosity and density μ and ρ and velocity v of the fluid approaching the plate. Determine a suitable set of π-terms to study this problem experimentally.

Pressure $- ML^{-1}T^{-2}$

Volume $=$

Bulk modulus $= ML^{-1}T^{-2}$

Velocity $= LT^{-1}$

Density $\rho = ML^{-3}$

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$D = f(w, h, \mu, \rho, v) \rightarrow$ general function relationship

$D = MLT^{-2}$

$w = L$

$h = L$

$\mu = ML^{-1}T^{-1}$

$\rho = ML^{-3}$

$v = LT^{-1}$

$(MLT^{-2})(L)^a(LT^{-1})^b(ML^{-3})^c = M^0L^0T^0$

$M \Rightarrow 1 + c = 0$

$L \Rightarrow 1 + a + b - 3c = 0$

$T \Rightarrow -b - b = 0$

$a = -2, \; b = -2$ and $c = -1$

$\Rightarrow$ The pi term becomes

$$\Pi_1 = \frac{D}{w^2 v^2 \rho}$$

The procedure is repeated with the second non-repeating variable h,

$$\Pi_2 = h w^a v^b \rho^c$$

$(L)(L)^a(LT^{-1})^b(ML^{-3})^c = M^0L^0T^0$

and

$M \Rightarrow c = 0$

$L \Rightarrow 1 + a + b - 3c$

$T = b = 0$

$\Rightarrow a = 1, \; b = 0, \; c = 0$

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$$\Pi_2 = \frac{h}{w}$$

The remaining non-repeating variable is μ

$$\Pi_3 = \mu w^a v^b \rho^c$$

with

$$(ML^{-1}T^{-1})(L)^a(LT^{-1})^b(ML^{-3})^c = M^0L^0T^0$$

$1 + c = 0$ (for M)

$-1 + a + b - 3c = 0$ (for L)

$-1 - b = 0$ (for T)

Solving the exponent we obtain

$a = -1,\ b = -1,\ c = -1$

$$\Pi_3 = \frac{\mu}{w v \rho}$$

Now, we require the three Π term, we should check if they are dimensionless

To check we use F, L, and T

$$\Pi_1 = \frac{D}{w^2 v^2 \rho} = \frac{F}{(L)^2 (LT^{-1})^2 (FL^{-4}T^{2})} = F^0L^0T^0$$

$$\Pi_2 = \frac{h}{w} = \frac{(L)}{(L)} = F^0L^0T^0$$

$$\Pi_3 = \frac{\mu}{w v \rho} = \frac{(FL^{-2}T)}{(L)(LT^{-1})(FL^{-4}T^{-2})} = F^0L^0T^0$$

10TH NOVEMBER 2021

n is no of variables

Buckingham's π Theorem

Since Rayleigh's method become laborious, when variable are more than fundamental dimensions (MLT) The difficulty is overcome by this method and this method state that:

1. If there are n-variables (independent and dependent variables) In a physical phenomenon are contain m-fundamental direction, the variables are thus arranged in (n-m) terms and this (n-m) terms are called π terms.

principles

1. Each π theorem is dimensionless and is independent of the system of unit.
2. Dimensional division or multiplication by a constant does not change the character of the π term.
3. Each π term contain (m+1) variables where m is the no of fundamental dimension and it is also called repeating variables

Solution (2)

fundamental variables MLT (3)

$D = f(\omega, h, \mu, \rho, v)$

$n = 6$

$m = 3$

$n - m$ = no of ~~Theorem~~ π terms

π terms $= 3$

π_1, π_2, π_3

$(\omega \ \rho \ v)$

$D = MLT^{-2}$

$\omega = L$

$h = L$

$\mu = ML^{-1}T^{-1}$

$\rho = ML^{-3}$

$v = LT^{-1}$

$\pi_1 = D\omega^{a_1} v^{b_1} \rho^{c_1}$

$= MLT^{-2} (L)^{a_1} (LT^{-1})^{b_1} (ML^{-3})^{c_1}$

$M \Rightarrow 0 = 1 + c_1$ --- (1)

$L \Rightarrow 0 = 1 + a_1 + b_1 - 3c_1$ ---- (ii)

$T \Rightarrow 0 = -2 - b_1$ ----- (iii)

$-b_1 = -2$

$-c_1 = -1$

substitute into eqn (2)

$1 + a_1 - 2 - 3(-1)$

$1 + a_1 - 2 + 3$

$a_1 = -2$

$$\Pi = \Delta\, \omega^{a_1} v^{b_1} \rho^{c_1}$$

$$= \Delta\, \omega^{-2} v^{-2} \rho^{-1}$$

$$\Pi_1 = \frac{\Delta}{\omega^2 v^2 \rho}$$

$$\Pi_2 = h\, \omega^{a_2} v^{b_2} \rho^{c_2}$$

$$M^0 L^0 T^0 = L (L)^{a_2} (LT^{-1})^{b_2} (ML^{-3})^{c_2}$$

$M \Rightarrow 0 = c_2$ ---- (i)

$L \Rightarrow 0 = 1 + a_2 + b_2 - 3c_2$ ---- (ii)

$T \Rightarrow 0 = -b_2$ ---- (iii)

$c_2 = 0$, $b_2 = 0$ } substitute into eqn (2)

$1 + a_2 + 0 - 3(0)$

$a_2 = -1$

$$\Pi_2 = h\, \omega^{a_2} v^{b_2} \rho^{c_2}$$

$$= h\, \omega^{-1} v \rho$$

$$\Pi_2 = \frac{h}{\omega}$$

$$\Pi_3 = \mu\, \omega^{a_3} v^{b_3} \rho^{c_3}$$

$$M^0 L^0 T^0 = ML^{-1}T^{-1} (L)^{a_3} (LT^{-1})^{b_3} (ML^{-3})^{c_3}$$

$M \Rightarrow 0 = 1 + c_3$ ---- (1)

$L \Rightarrow 0 = -1 + a_3 + b_3 - 3c_3$ - (2)

$T \Rightarrow 0 = -1 - b_3$ -- (3)

$c_3 = -1$, $b_3 = -1$ } substitute into eqn 2

$1 + a_3 - b_0 - 3(-1)$

$0 = 1 + a_3 - 1 \cancel{} - 3 - 1$

$a_3 = -1$

$\pi_3 = \mu \omega^{a_3} v^{b_3} \rho^{c_3}$

$\mu \omega^{-1} v^{-1} \rho^{-3}$

$$\pi_3 = \frac{\mu}{\omega v \rho}$$

$\pi_1 = \phi(\pi_2, \pi_3)$

$$\frac{D}{\omega^2 v^2 \rho^2} = \phi\left[\frac{h}{\omega}, \frac{\mu}{\omega v \rho}\right]$$

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